United States Department of Agriculture

AGRICULTURAL MARKETING SERVICE

A NEW TECHNIQUE
FOR THE
ESTIMATION OF CHANGES IN FARM EMPLOYMENT

NUMBER I

OF A SERIES OF

ANALYSES OF SAMPLE FARM DATA

Prepared with the assistance of the Work Projects Administration
for the City of New York
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This study was conducted by the Bureau of Agricultural Economics in 1938-39. With the establishment of the Agricultural Marketing Service on July 1, 1939, the work (together with the personnel named) was included among the functions transferred to the latter agency of the Department of Agriculture.

The assistance given by the Board of Education of the City of New York in the publication of this report is gratefully acknowledged.

EXPLANATORY NOTE

This report gives the results of an investigation to determine more exactly the nature of the voluntary sample of employment conditions on farms of regular crop correspondents of the Agricultural Marketing Service of the United States Department of Agriculture. A basis for improving quantitative estimates of employment based on that voluntary sample was developed from this investigation. Analysis of the basic data, made possible by this study, uncovered a method for stratifying and weighting the sample which overcame many previous difficulties. The new method was thoroughly tested during the study, and the feasibility of using a voluntary sample in estimating employment was demonstrated.

Several weak links remain, however. No attempt was made to analyze the most important deficiency—that of annual estimates of the total number of farms. Existing figures, chiefly for census periods, had to be used. These data on farm numbers are essential in any attempt to estimate employment from a sample. The sample approach makes it necessary first to estimate per-farm employment which is then built up to total employment by using the number of farms. The lack of adequate data on numbers of farms by years and by States seriously handicapped the results obtained but it is believed not to have vitiated conclusions with regard to the basic method.

This study was undertaken in October 1938 with the assistance of the Works Progress Administration of New York City (now Work Projects Administration) Official Project No. 765-97-3-16 and was completed in July 1939. The work was done under the general supervision of C. F. Sarle, Principal Economist, and R. F. Hale, Senior Statistician, both of the Agricultural Marketing Service. Glenn D. Simpson, Associate Statistician, representing the Agricultural Marketing Service on several phases of agricultural research, was largely responsible for the smooth operation of the administrative details on the project. C. B. Lawrence, Jr., Coordinator of Statistical Projects, of the Work Projects Administration furnished many helpful suggestions and criticisms.

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A NEW TECHNIQUE FOR THE ESTIMATION OF CHANGES IN FARM EMPLOYMENT

By A. R. Sabin, Associate Agricultural Statistician

The problem of making current estimates of agricultural employment has become increasingly important in recent years. Present data regarding employment on farms furnish no more than rough approximations of the actual changes in the level of agricultural employment in any given State. This has seriously handicapped efforts to work out objective formulae for the distribution of relief funds and a more definite method for estimating the actual relief load by States.

Farm workers constitute one of the largest groups still outside the provisions of various social security laws. One difficulty in making security provisions for this group has been the lack of sufficient information regarding the year-to-year and within-season changes in the distribution of the number of workers so employed.

In January 1935, the Bureau of the Census counted 12,408,000 persons working on farms. Of this total, 10,762,000 were classified as family workers and 1,646,000 as hired laborers. The 1935 census figures on family workers are not comparable with those found in previous censuses in most States, but the figures on distribution of family workers in 1935 appear to be more nearly in agreement with data obtained from the crop reporters of the Department of Agriculture than in the case of figures on hired labor. After adjusting for lack of comparability with previous censuses of occupations, the National Research Project of the WPA[1] estimated 9,175,000 workers made up of 7,482,000 family workers and 1,693,000 hired help as of January 1, 1935. Information obtained in January of one year, however, is not sufficient to furnish a basis for solving current problems. Farming is a highly seasonal occupation and January is near the low point of the year in farm

1. Shaw, E. E., and Hopkins, J. A. Trends in Employment in Agriculture, 1909-36. U. S. Works Prog. Admin., Philadelphia, Rpt. A-8, 163 pp., illus. November 1938. (See pp. 6 and 7; also Appendix A, p. 86 et seq.; and p. 153.)

activity and employment in most States. Furthermore figures for a single date once in five or ten years are of little help in attacking the problem of measuring annual changes in the level of employment.

Monthly Employment Data

Comprehensive monthly data regarding changes in agricultural employment have been collected since 1924 by the United States Department of Agriculture. About 20,000 questionnaires are returned to the Agricultural Marketing Service around the first of each month by its regular crop correspondents.

The Department did not collect any information on farm labor other than wage rates until 1918. At that time, questions were added to the regular monthly crop schedule asking:

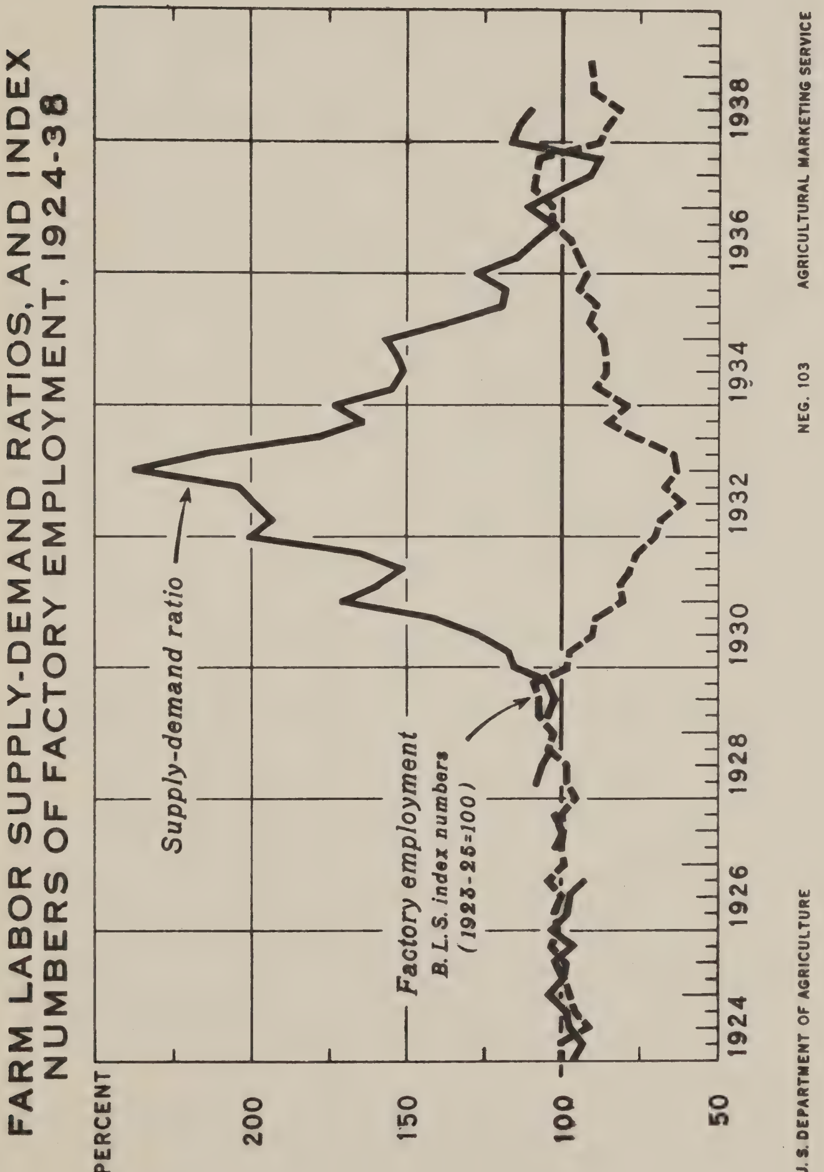
(1) Supply of farm labor in percent of normal, and

(2) Demand for farm labor in percent of normal.

These questions had reference to the locality with which the reporter was familiar and replies were based only on the reporter's judgment. Despite the subjective nature of the questions, the replies proved of real value as indicators of the need for hired help and the availability of persons to fill this need. A supply-demand ratio calculated from these data clearly showed changes in the general conditions affecting supply of and demand for farm labor. Compared with indexes of factory employment an inverse relationship was obtained (fig. 1).

A supply-demand ratio, however, is indicative only of a surplus or deficiency of workers available for hire in rural areas and does not provide an indication of changes in the number of persons actually hired by farmers. Increased demand does not necessarily result in increased employment because reduced farm income or other economic

Figure 1.—Farm labor supply-demand ratios, and index numbers of factory employment, 1924-38.



conditions may prevent the hiring of needed help. Changes in supply are not reflected in employment figures because it cannot be assumed that all persons available for farm work are actually employed at that work. Furthermore, the subjective nature of the questions may result in some bias in that supply and demand tend to be estimated as complements of each other rather than in percent of normal. An increasing demand generally seems to be associated with a decreasing supply, and vice versa, quite apart from the normal or average situation.[2]

An objective approach to the employment problem was desirable. In an effort to obtain a more precise indication of changes in farm employment, additional questions were added to the schedule asking for the actual numbers of family and hired workers employed on farms of crop reporters. These questions were added to the schedule in October 1923 and have been continued with little change to the present time. The data were summarized and published in the form of numbers employed per hundred farms of crop reporters, broken down to show family and hired workers separately.

Selectivity of the Sample

Adoption of this individual farm approach still left a number of problems. Crop-reporter farms were found to be highly selective. In April 1930, for the United States as a whole, crop correspondents reported an average of 238 family workers per 100 farms as compared with 122 found by the census on the same date. The respective figures for hired workers were 92 and 45. In January 1935, crop correspondents averaged 205 family workers per 100 farms compared with 158 reported by the census. On the same date, crop correspondents hired 77 workers per 100 farms, while the United States average of census enumerations was only 24.

2. Social Science Research Council; Advisory Council on Social and Economic Research in Agriculture. Research in Farm Labor. Social Sci. Res. Council, Bul. 16, 84 pp., illus. New York. June 1933.

Black, J. D. Agricultural Wage Relationships. Rev. Econ. Statis. 18: 8-15. February 1936.

Average employment per farm, however, varied widely among States. In 1935 the census, taken in January, found a range in family labor from 1.10 workers per farm in Connecticut to 2.15 in Alabama. Hired workers ranged from 0.10 in South Dakota to 1.11 in Arizona.

Table 1.—Percentage of farms employing specified numbers of unpaid family workers, January 1, 1935

FARMS OF CROP REPORTERS[1]

DIVISION	Number of workers per farm			
	0	1	2	3 & over
	Pct.	Pct.	Pct.	Pct.
New England	3.2	58.4	30.9	7.5
Middle Atlantic	1.8	50.2	35.5	12.5
East North Central	.9	49.5	36.6	13.0
West North Central	.5	48.6	35.2	15.7
South Atlantic	1.4	37.8	32.4	28.4
East South Central	1.7	32.8	32.7	32.8
West South Central	9	38.5	34.8	25.8
Mountain	1.0	50.1	32.6	16.3
Pacific	6.4	54.1	28.9	10.6
United States	1.4	46.1	34.2	18.3

ALL FARMS (CENSUS OF AGRICULTURE)

New England	9.9	67.0	17.2	5.9
Middle Atlantic	6.4	65.2	20.3	8.1
East North Central	3.5	65.0	22.7	8.8
West North Central	3.1	63.3	23.1	10.5
South Atlantic	5.9	52.5	21.7	19.9
East South Central	4.3	52.0	24.0	19.7
West South Central	3.8	55.1	22.4	18.7
Mountain	5.8	65.6	19.0	9.6
Pacific	11.1	65.9	16.6	6.4
United States	4.8	59.9	22.1	14.2

1. Jan. 1, 1936 data were used for Michigan and Oklahoma.

Table 2.—Percentage of farms employing specified numbers of hired workers, January 1, 1935

FARMS OF CROP REPORTERS[1]

DIVISION	Number of laborers per farm					
	0	1	2	3	4	5 & over
	Pct.	Pct.	Pct.	Pct.	Pct.	Pct.
New England	47.9	31.1	11.7	4.8	1.3	3.2
Middle Atlantic	57.2	29.5	8.8	2.3	1.3	.9
East North Central	68.6	24.7	4.7	1.0	.6	.4
West North Central	75.1	19.8	3.8	.9	.2	.2
South Atlantic	52.4	25.4	10.7	4.6	2.4	4.5
East South Central	61.6	20.1	8.6	3.7	2.5	3.5
West South Central	79.2	13.4	4.7	.8	.5	1.4
Mountain	76.8	15.5	4.3	1.8	.6	1.0
Pacific	53.3	23.3	9.8	4.3	2.1	7.2
United States	67.1	21.8	6.3	2.1	1.0	1.7

ALL FARMS (CENSUS OF AGRICULTURE)

New England	76.5	16.9	3.8	1.2	.6	1.0
Middle Atlantic	76.5	18.3	3.3	.9	.4	.6
East North Central	83.8	13.7	1.7	.4	.1	.3
West North Central	86.8	11.2	1.3	.3	.1	.3
South Atlantic	83.8	11.0	2.8	1.0	.5	.9
East South Central	91.7	5.9	1.3	.4	.3	.4
West South Central	89.3	7.2	1.8	.6	.4	.7
Mountain	85.5	9.7	2.4	.9	.5	1.0
Pacific	78.5	13.7	3.6	1.4	.8	2.0
United States	85.8	10.6	2.0	.6	.3	.7

1. Jan. 1, 1936 data were used for Michigan and Oklahoma.

It will be seen from tables 1 and 2 that few farms are without any family labor. Most of them are operated by from 1 to 3 members of the farm family. On the other hand, relatively few farms hire any labor during the winter period. The census reported that only 14 percent of all

farms in the United States hired any labor at all on January 1, 1935, and only 2 percent of all farms hired more than 2 workers at that time.

The proportionate distribution of farms employing different numbers of workers on any one specific date also shows a wide State-to-State variation. The distribution of farms according to the numbers of workers on January 1, 1935, shows a striking difference between family and hired labor, both for the census and for the sample. Considered separately family and hired labor both showed significant differences as between the census and the sample distribution (fig. 2).

Crop reporters generally average fewer farms having none or one family worker and more farms with 2 or more than is the case with all farms as reported by the census. This selectivity with respect to farm employment results from the character of the crop-reporter lists which include a relatively heavy proportion of larger-than-average farms, having more-than-average labor requirements. Similarly, a larger-than-average proportion of crop reporters hire some labor. For the United States, 33 percent of crop-reporter farms hired some labor in January 1935, whereas according to the census only 14 percent of all farms did so.

The selective nature of the sample suggests the possibility that seasonal variations in employment, calculated from these data, differ from the pattern for all farms. Information is not available for determining this difference with any precise degree of accuracy, but numerous special investigations in different type-of-farming areas show considerable variation as between farms of crop reporters and farms covered by the investigations mentioned. The difference appears clearly in the following table for New Jersey and North Carolina.[3]

3. Folsom, Josiah C. Farm Labor Conditions in Gloucester, Hunterdon, and Monmouth Counties, New Jersey, April-May, 1936. U. S. Bur. Agr. Econ. 51 pp. February 1939. (Mimeographed.) (The manuscript giving data for North Carolina is in preparation.)

Figure 2.—Percentage distribution of farms classified by number of workers, United States, Jan. 1, 1935.

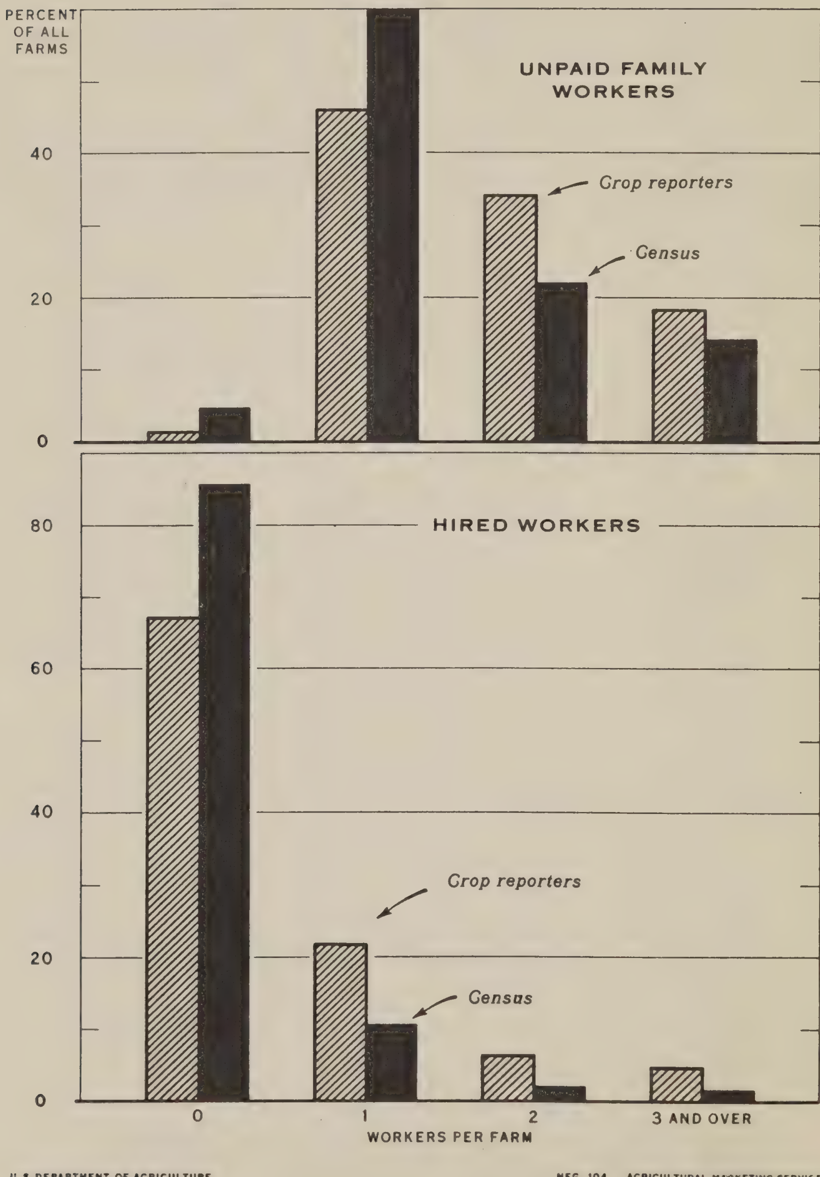


Table 3.—Relative seasonal changes as indicated by returns from crop reporters and special farm employment surveys, New Jersey and North Carolina, 1935
(January 1 = 100)

Month	New Jersey				North Carolina			
	Family		Hired		Family		Hired	
	Crop reporters	Survey	Crop reporters	Survey	Crop reporters	Survey	Crop reporters	Survey
January	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
February	97.3	100.3	103.2	101.5	97.4	103.8	92.6	113.1
March	90.7	101.9	116.0	132.6	102.0	115.2	94.1	355.7
April	98.4	104.1	120.2	170.5	100.7	123.3	79.3	398.1
May	95.6	104.8	127.7	195.9	99.3	154.4	85.2	546.5
June	95.1	107.9	121.3	237.2	118.7	182.9	118.5	800.2
July	104.9	111.2	130.8	296.9	138.7	181.6	124.4	626.5
August	96.2	111.0	142.6	321.6	128.9	160.2	123.7	450.5
September	89.6	109.8	144.7	271.4	129.5	166.5	116.3	209.2
October	102.7	103.7	106.4	236.9	122.3	169.0	108.9	168.2
November	101.1	101.1	138.3	156.4	128.2	158.8	140.0	134.2
December	91.8	99.9	95.7	112.9	100.3	125.9	108.1	99.1

Source: The surveys, relating to employment during 1935, were conducted by the WPA in 1936, in cooperation with the U. S. Department of Agriculture. In New Jersey, 2025 farms were included; in North Carolina, 1,769 farms. Crop-reporter data are numbers of workers per 100 farms on farms of crop reporters.

Errors due to the selectivity of the sample can be partly corrected by proper stratification and weighting. If the seasonal movements within strata differ as between the sample and the universe, however, full adjustments cannot be obtained by this method. This is the case with crop-reporter farms which are more likely to be year-around farms having smaller seasonal variations in employment than is the case with other farms.

The sharecropper situation in Southern States provides many difficulties in estimating agricultural employment, as in the preparation of all agricultural statistics for those States. Few sharecroppers hire labor at any time, but large family employment is usual. Each sharecropper unit is reported as a farm by the census. In the generally accepted terminology these sharecropper units are merely parts of a farm or plantation and not separate farming units. Sharecroppers are under-represented in the crop-reporter sample and are frequently reported by the plantation owner either as family or hired workers according to local custom. Strictly speaking they are neither, and deserve a special category

of their own. Inclusion of sharecroppers as farm operators in census data results in an unusually large proportion of farms having 2 or more family workers. Since they do not actually receive a money wage, a large proportion of farms hire no labor in States where this form of tenancy is common.

In the Southern States 716,000 sharecroppers were listed as farm operators, or about 21 percent of all operators found by the census in these States in 1935. Seven States showed more than the average percentage of sharecroppers with Mississippi the highest at 44. The variation between States, shown in table 4 makes for large State-to-State differences in comparability of crop-reporter and census data in addition to differences noted earlier.

Table 4.—Percent of all farms operated by sharecroppers, Southern States, January 1, 1935

State	Percent	State	Percent	State	Percent	State	Percent
Del.	2.6	N. C.	22.1	Ky.	11.9	Ark.	25.9
Md.	3.7	S. C.	27.9	Tenn.	18.8	La.	29.5
Va.	9.2	Ga.	32.1	Ala.	24.9	Okla.	6.4
W. Va.	2.9	Fla.	6.4	Miss.	43.9	Texas	15.3

Average all Southern States = 20.9

Source: Census of Agriculture, 1935.

Estimation of Changes in Farm Employment

Beginning in 1923 estimates were made of the total number of workers employed on farms of crop reporters. The data were classified to show family and hired labor separately. These estimates were based on simple averages of data for farms of crop reporters and were expressed in numbers per 100 farms.

Despite certain serious limitations of this approach, facilities were not available for further analysis. It was necessary therefore to continue publication of these data inasmuch as they were the only figures available on farm employment. Seasonal changes in employment, calculated from crop-reporter averages, generally were unreasonably small compared with actual labor requirements on farms. Furthermore, there were long intervals between census check points, and data on changes in the numbers of farms were inadequate. Chance fluctuations in the size and distribution of the sample also were difficult to appraise with existing facilities.

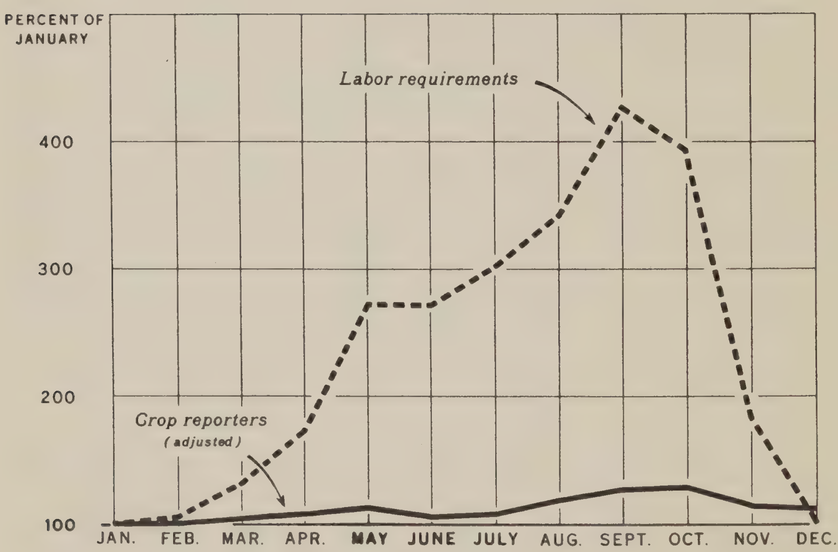
It was quite generally recognized that new procedures had to be developed in order to obtain more accurate estimates of changes in farm employment, as well as better to approximate the actual level of employment. In 1936, the National Research Project of the WPA on Reemployment Opportunities undertook an analysis of available information on farm employment and prepared employment estimates for the period, 1909-36. The basic procedure involved the computation of seasonal adjustment factors which were applied to crop-reporter averages; and the whole series was then equalized with census check points.[4] Also considered in that study were: adjustment of census data to comparable bases; estimates of changes in farm numbers; and, long-time changes in farm labor requirements.

Some difficulties were encountered in the use of that method, however, which led to further analysis. It was difficult to obtain accurate estimates by using a normal seasonal adjustment factor during a year when unusual seasonal conditions, such as drought, prevailed. Supplementary adjustments, therefore, were necessary. Chance fluctuations in sampling also were troublesome, particularly with respect to individual State data.

4. Shaw, E. E., and Hopkins, J. A. See pp. 86 et seq. of reference cited in footnote 1, p. 1.

Even in normal years the use of this method was unsatisfactory for some States as will be seen from the accompanying chart for California. Two indications of seasonal change are shown: (1) crop-reporter data adjusted according to the procedure worked out by the 1936 project, and (2) estimated farm labor requirements in the principal agricultural counties.[5]

Figure 3.—Hired farm workers, California, 1935—Comparative indications of seasonal change.



5. California State Relief Administration. Survey of Agricultural Labor Requirements in California, 1935. 253 pp., illus. December 1935. (Relatives derived from data in table 3, p. 21.)

Reanalysis of Monthly Reports on Farm Employment

The need continued for eliminating some of the difficulties noted above and, if possible, for preparing a method whereby changes in agricultural employment could be estimated directly from the sample. For the period, 1929-37, all available crop-reporter schedules, numbering about 2,500,000, were assembled, edited and sorted. The facilities of the Works Progress Administration were then used to list reports on employment for each crop-reporter farm.

Matched sample tabulations were first considered as a method of minimizing the effects of chance fluctuations in the size and distribution of the sample. Matching was done both on a month-to-month basis, from which link relatives were computed, and from given month to base month. In matching samples from month to month, however, a strong upward bias was detected although many random fluctuations were eliminated. Examination indicated that this bias, inherent in replies to mailed schedules, resulted from a tendency on the part of voluntary crop reporters who hire considerable help to cease reporting when employment started on the down-grade.

Experimental work soon demonstrated, however, that this upward bias disappears when current reports are matched to a given base month each year.

Ratios of change computed from simple averages of matched samples, however, showed little more seasonal variation than was the case with simple averages of unmatched data. Furthermore, the matching process does not correct for selectivity in the basic data. In order to assign the proper influence to farms having different labor requirements, it was necessary to stratify and weight the sample data. A breakdown of January 1935 census employment figures provided the only data showing the number of farms in each State reporting specified numbers of workers. Crop-reporter farms were then stratified each year according to the numbers of workers employed on January 1, and separate monthly averages were computed for each stratum as shown in the following table:

Table 5.—California Farm Employment: Worker groups and proportion of farms with specified numbers and hired workers, January 1, 1935, and per-farm employment of hired hands by months and groups, 1937

Hired workers per farm Jan. 1, 1935		Hired workers employed per farm, 1937											
Groups	Farms reporting	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	Percent	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.
0	69.7	0.0	0.3	0.4	0.4	0.5	0.6	0.8	0.8	1.4	1.7	0.7	0.2
1	18.2	1.0	1.2	1.1	1.0	1.2	1.6	1.4	1.8	2.2	1.9	2.0	1.0
2	5.2	2.0	2.1	2.2	2.3	2.7	2.7	3.4	2.8	4.7	5.3	4.5	1.9
3	2.1	3.0	3.4	3.6	2.2	2.4	3.4	4.0	6.9	6.3	4.7	4.2	2.3
4	1.3	4.0	4.3	3.8	4.0	4.3	5.1	6.3	8.2	5.5	3.0	5.0	3.0
5	.8	5.0	3.8	3.5	4.0	4.6	6.7	6.1	6.7	12.0	11.6	7.8	4.4
6	.6	6.0	6.1	5.8	5.2	6.3	6.0	6.4	7.3	8.0	13.2	8.8	6.0
7	.3	7.0	8.4	7.6	7.0	7.6	8.0	8.2	9.6	8.0	14.4	10.8	6.6
8	.3	8.0	6.5	6.5	4.2	3.2	3.8	4.5	14.0	17.2	11.0	3.2	2.0
9	.2	9.0	5.3	5.7	6.0	6.0	6.3	8.8	10.0	9.0	7.0	8.8	10.0
10-over	1.3	21.9	19.5	22.6	22.6	23.4	23.9	26.4	25.7	25.1	30.2	22.6	21.5
State	100.0	.825	1.040	1.129	1.081	1.237	1.436	1.641	1.805	2.388	2.599	1.708	.904

Source: 1935 data from the Bureau of the Census. 1937 data from U. S. Department of Agriculture.

Each stratum was weighted by the number of farms in its respective group as reported by the census, and average per-farm figures obtained for each month.[6] Month-to-month ratios of change, computed from monthly average, were applied to estimated employment figures[7] for January 1935 to obtain comparable average-per-farm employment for the whole series. Per-farm employment in each State was multiplied by the estimated number of farms[7] in arriving at estimates of total employment.

The problem of obtaining proper adjustment for year-to-year changes was attacked by carrying the tabulation each year into January of the following year. If some adjustment of this nature is not made, the January figure each year will revert to the level established by the census in January 1935. By obtaining 13-month data it was possible to compute January-to-January changes in employment per farm, from which relatives were derived. The overlapping of relatives, computed in this manner, permitted calculation of a continuous series without a break at the beginning of each year.

Results of the Reanalysis

A large part of the discussion herein is occupied by consideration of the hired labor situation. This is done, despite the much larger proportion of unpaid family workers to total workers on farms, for several reasons:

1. The seasonal variation in employment of hired workers is much greater than for family workers.

2. The farm-to-farm range in the numbers of hired workers is much greater than for family workers, making it more difficult to obtain accurate estimates.

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6. As of January average for each stratum (that is, 0, 1, 2, 3, etc.) is fixed the averages for subsequent months necessarily are matched with January eliminating the upward bias found in month-to-month matchings.

7. Shaw, E. E., and Hopkins, J. A. See footnote 1, p. 1.

3. The breakdown of farms reporting specified numbers of family workers, used to weight the different strata, was not carried far enough by the census to provide as detailed results as in the case of hired workers.

Examination of figure 5 shows a smaller seasonal variation in the number of family workers, computed by stratifying and weighting the data, than was obtained by the use of seasonal factors based on labor requirements. Part of the difference may be a result of the inadequate breakdown of census figures available for weighting. Most of the difference, however, probably results from the difference in definition between employment and labor requirements, discussed in the following section.

Employment vs. Labor Requirements

Confusion sometimes arises with respect to discrepancies between labor requirements for a given crop or area and the labor actually employed. Frequently the labor requirements at peak seasons are found to be much higher relative to slack seasons than is the case with numbers of workers employed. The apparent discrepancy results from the fact that the two phenomena are inherently different although frequently they are assumed to be almost synonymous, or at least to show proportionate change.

Labor requirements commonly are computed in man-days, or some other convenient unit, necessary to produce a given crop or to operate a given farm. Employment, on the other hand, is computed as the number of persons working 2 or more days per week required to produce the crop or operate the farm. A farm with labor requirements of only 2 man-days per week during the winter might very easily require 12 man-days per week in mid-summer. Thus, an increase of 500 percent in man-days worked would result in only 100 percent increase in the number of men

employed per week. Similarly, a decrease in the amount of labor required often is not fully reflected in employment figures.

Labor requirements generally do not provide a satisfactory basis for estimating employment at the present time. The difference in definitions is one reason. Another is the inadequacy of data on labor requirements. For the most part surveys on labor requirements are rather widely scattered and confined to special crops or small geographical areas. Usually they are confined to a limited objective such as the growing of a specific crop in a given area. Even more important is the fact that rarely is any separation made of the part of the crop grown largely by the use of hired workers and the part produced largely or entirely by family workers.

Per-acre labor requirements for a given crop may be identical for family-sized and commercial-sized units. Employment, however, varies relatively little on the former compared with commercial units where most of the necessary labor is hired. Even in commercial areas a large part of farm production is grown on family-sized farms on which labor is rarely or never hired. Furthermore, labor requirements vary with changes in the degree of mechanization of the farm business. Present data are inadequate to provide current information on changes in labor requirements. Present data also are inadequate to measure changes in current relationships between labor requirements and employment, particularly on the family-sized farm. The effect of some of these differences is illustrated by figure 4 which gives a comparison of labor requirements and employment in California in 1935.

Estimates of employment prepared by the new method appear reasonable in the light of other available information. Figures 5 and 6 give a comparison of estimates for the Middle Atlantic States prepared by the old method and by the proposed new method.

Source: Labor requirements from California State Relief Administration. Employment from U. S. Department of Agriculture estimates.

Figure 4.—Labor requirements and employment of hired farm workers, California, 1935.
(January = 100)

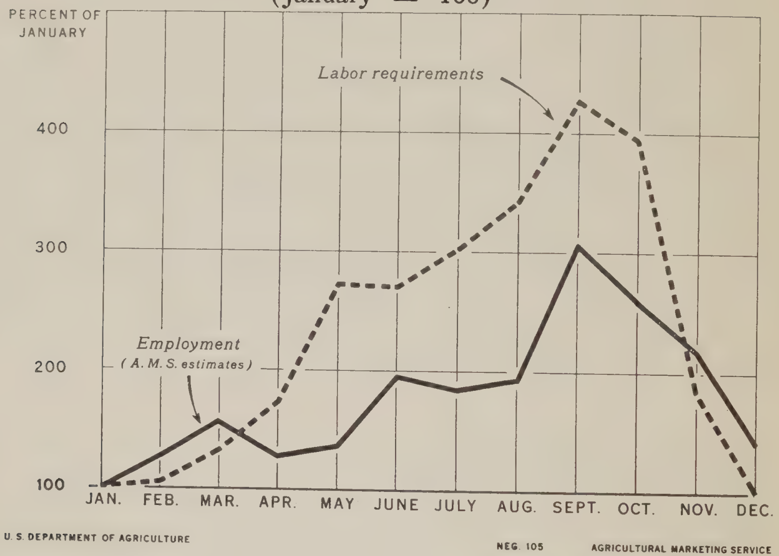


Figure 5.—Unpaid family workers employed on farms, Middle Atlantic States, 1934-37.

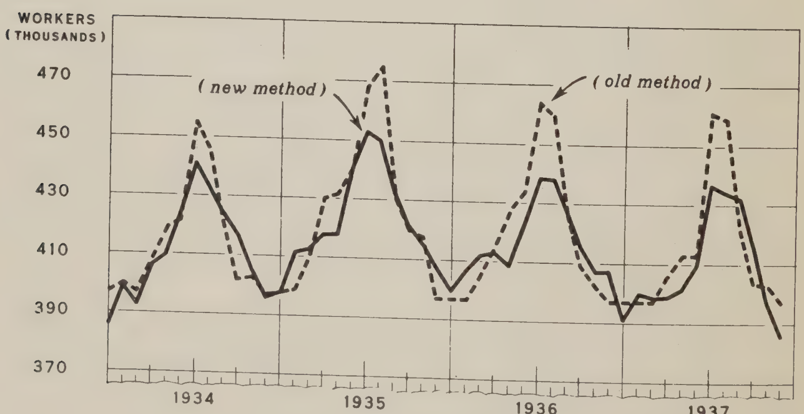
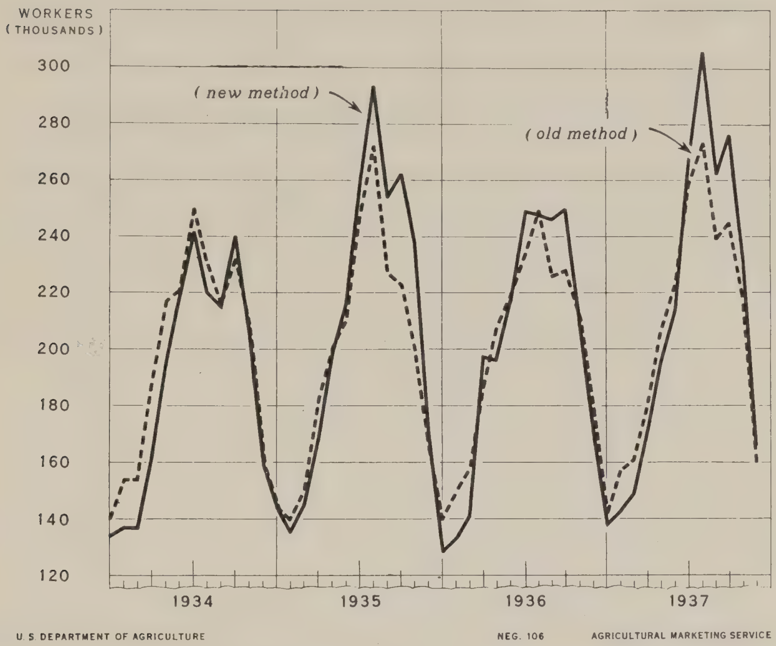


Figure 6.—Hired workers employed on farms, Middle Atlantic States, 1934-37.



It will be noted that the two methods agree rather closely as regards direction of seasonal movements but that the degrees of the movements differ. This difference is owing, in part at least, to the difficulty of computing separate monthly adjustment factors for family and hired workers in compiling the old series. Seasonal factors cannot be applied equally to family and hired labor indications, and breaking down the seasonal factor may result in an inequitable distribution between the series for family and hired workers.

It seems probable in some cases that the most accurate estimates will lie somewhere between the old method, based on adjustments of reported figures more in line with seasonal labor requirements, and the new method. In stratifying and weighting crop-reporter data according to the new method, it must be remembered that the sample in some cases is composed of farms tending toward the minimum of seasonal variation as to labor employed. On

the other hand, as was noted above, seasonal variation in employment based on labor requirements is likely to be exaggerated.

Limitations of the Stratified-Sampling Method

The method of stratification developed by this study is subject to a number of limitations that are unavoidable in the use of mailed schedules and voluntary returns. In the main these limitations are:

1. Difficulty of obtaining representative samples,

2. Inadequate numbers of reports,

3. Misunderstanding of the questions,

4. Irregularity in reporting,

5. Defections in the sample from one year to another.

Other limitations that apply both to the voluntary sample and to other methods that may be used are:

1. Census check data are available as a basis for weighting only at infrequent intervals.

2. Numbers of farms are not estimated currently.

3. It is often difficult to distinguish between employment and unemployment particularly with respect to farm family workers.

From the information available it is not certain that a group of crop reporters using a specified number of workers, either family or hired, on January 1, is representative of all farms in that category. In the absence of more detailed information, however, it has been necessary to assume representativeness in order to prepare estimates by this method. The voluntary nature of the replies results in

other difficulties. It is always difficult, and frequently impossible, with a mailed questionnaire to obtain enough reports to get any required degree of stability for the different strata, particularly the less populous ones. Misunderstanding of the questions and irregularity in reporting will always present some difficulty in the use of any mailed inquiry. Defections in the mailing list are of sufficiently frequent occurrence that the number of monthly returns matching with those of the base month declines steadily. As a result, the analysis becomes less valid after the lapse of a year. This necessitates re-stratification each year according to strata set up from the previous census. Consequently year-to-year changes are not estimated as accurately as would be desired. In the situation currently under consideration the last census was taken January 1, 1935. The next census will be taken in 1940, and as of April 1, but it may well be 1941 or 1942 before data are available for weighting employment estimates. Large shifts in employment frequently occur in 6 or 7 years. Differences in dates of census enumerations furnish an additional source of difficulty.

Limitations of other current methods of estimating agricultural employment have been noted above. The relatively long period between censuses does not allow an opportunity to correct for underenumeration or for faulty or noncomparable wording of the questions, which is essential in any base point or bench mark for current estimates. Large shifts also occur from census to census in the actual numbers employed as well as in the distribution of these workers among farms. Estimates, therefore, become more liable to error as they get farther from their base points. Numbers of farms, likewise, are estimated from census data and suffer from these same limitations. In this case both the definition of a farm and the actual number contribute difficulties. Considerable difference exists between the census definition of a farm and that understood by crop reporters, particularly in the South. Also, the census is limited by a degree of incompleteness that is not readily calculable on any given data. The difficulty of drawing the

line between employment and unemployment, particularly with respect to farm family workers, is evident in any method for estimating farm employment.

Suggested Improvement in Farm Employment Estimates

In connection with the improvement of farm employment estimates several courses may be considered:

1. Further improvement in representatives and accuracy of the voluntary sample,

2. Sampling by other means, such as enumerated schedule,

3. A combination of these methods.

Improvement in estimates derived from mailed inquiries could be made by matching reports to a base month and stratifying and weighting the data according to census classifications, as previously outlined. This would necessitate additional funds and facilities for the expanded work. It would also be desirable that more adequate census data be obtained. This is particularly necessary with respect to some further breakdown of family labor items. Additional classifications to show the numbers of farms operated by 3, 4, 5, and over 5 workers would be very helpful.

A separation of sharecropper families from other operators would be of great benefit in reporting data from the Southern States. An even more significant breakdown probably could be made on the basis of type-of-farm or some other significant factor relating to labor requirements. Other necessary additions to present procedure include strengthening the mailing lists. The selective nature of the present sample, noted earlier, could be overcome,

in part, by building up special lists in specialized areas or by making other arrangements to obtain the data. Coverage in areas of specialized fruit, truck crop and livestock production probably could be greatly expanded and a fairly good representation obtained by use of a mailed inquiry. Relatively, a large proportion of crop-reporter farms that hire large numbers of workers are in these special categories. Practically, however, the actual number of reports on such farms in the sample is too small in most States to derive stable averages for that group. On the other hand, it is doubtful whether representative data for subsistence, seasonal, or part-time farming can ever be obtained by mail. The building up and maintenance of mailing lists are of primary importance. Follow-up schedules should be used and replacements made constantly to maintain a reasonable degree of precision in each of the different strata.

The large lists of general farmers now maintained and the basic work already performed in the field of farm employment statistics by the Department of Agriculture, provides a logical framework for the desired improvement in these employment estimates. Many special lists, or facilities for obtaining such lists, are also available. An increase in clerical personnel would be necessary, however, to take care of additional work involved in maintaining more stable lists and in listing, stratifying, and weighting the data. Additional technical personnel would be required to supervise the work, to assemble the data, and prepare estimates.

An enumerative sample could be taken in such a way as to eliminate many sources of error that are inherent in a voluntary sample. If such sampling is done with blocks of farms or small areas as sampling units, it should be possible to obtain data both on month-to-month and year-to-year changes in employment on farms as well as on changes in numbers of farms. Other valuable information also could be obtained regarding labor requirements, movement of

farm population, standard wage and piece-work wage rates, unemployment, etc. Information on crop and livestock items could be gotten and used to forecast farm employment requirements. Employment in rural nonfarm occupations could be readily obtained as well as urban nonindustrial employment in smaller towns and cities.

In many instances a combination of voluntary and enumerated samples may be advisable. This could be effected by strengthening mailing lists insofar as possible and restricting enumeration to those groups not readily sampled by mail. Such a combination of voluntary and enumerated samples would certainly be better than the straight mailed inquiry now in use. But an enumerated sample is believed to be superior to either of the other two methods and likely to supply the most precise indications of changes in the actual level of farm employment.

Conclusions

Although a number of questions regarding the estimation of farm employment are left unanswered, enough evidence has been unearthed during the course of the investigation to support the following conclusions:

(1) The accuracy of estimates based on a voluntary mailed sample can be greatly increased by stratification and weighting of the data.

(2) The use of labor requirements alone for the estimation of changes in farm employment, or seasonal fluctuations in labor requirements as a basis for adjusting reported data, is likely to result in considerable error in many cases by indicating seasonal changes in employment that are larger than those actually taking place.

(3) The use of sample data from a single stratum, such as the largest farms, will not give an accurate basis for estimating changes in total farm employment owing to the different rates of seasonal change on farms with large and small labor requirements.

(4) The difficulty of obtaining adequate sample information by mail for certain strata of farmers, such as sharecroppers and nonresident operators, makes it advisable to explore other methods of sampling. In this connection the use of an enumerated sample is suggested.

United States Department of Agriculture

AGRICULTURAL MARKETING SERVICE

THEORETICAL ASPECTS OF THE USE OF THE CROP METER

NUMBER II

OF A SERIES OF

ANALYSES OF SAMPLE FARM DATA

Prepared with the assistance of the Work Projects Administration
for the City of New York

O. P. No. 765-97-3-16

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The assistance given by the Board of Education of the City of New York, in the publication of this report is gratefully acknowledged.

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New York City

EXPLANATORY NOTE

This report gives the results of an investigation to determine the validity of assumptions made in translating linear measurements of crop frontages on highways into estimates of corresponding crop acreages in the region traversed. Such a study must obviously be based upon a universe of known constitution so that various phases of the problem can be examined in detail. Agricultural data obtained from usual sources are not sufficiently extensive for an analysis of this kind but aerial survey photographs made available by the Agricultural Adjustment Administration provided a good source of experimental material for the purpose at hand. After the crops in the various individual fields shown on the photographs were identified by visits to the farm operators concerned and the highways traversing the region were traced on the photographs, an ideal universe for study was made available.

This study was undertaken in November 1938 by the Bureau of Agricultural Economics with the assistance of the Works Progress Administration of New York City (now Work Projects Administration) as Official Project No. 765-97-3-16 and was completed in June 1939. With the establishment of the Agricultural Marketing Service on July 1, 1939, the work was transferred to that agency of the Department of Agriculture. The study was made under the general supervision of C. F. Sarle, Principal Economist, and A. J. King, Agricultural Statistician, both of the Agricultural Marketing Service. Glenn D. Simpson, Associate Statistician, representing the Service on several phases of agricultural research, was largely responsible for the administrative details of the project. C. B. Lawrence, J., Coordinator of Statistical Projects of the Work Projects Administration, furnished many helpful suggestions and criticisms.

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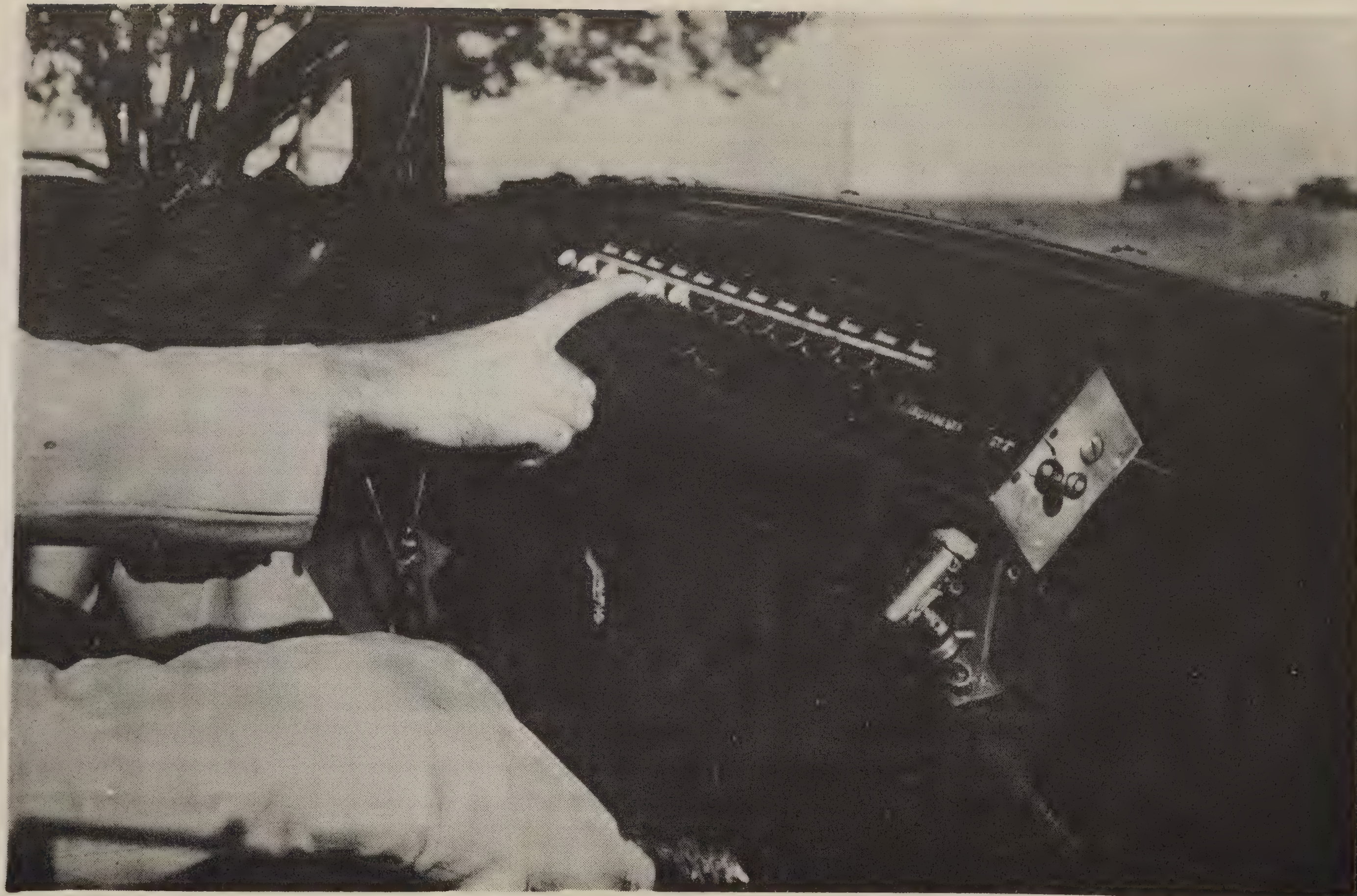

Figure 1.—Crop meter in operation. Small dials register frontages of different crops and large dial in lower right-hand corner registers total distance driven.

THEORETICAL ASPECTS OF THE USE OF THE CROP METER

By Walter A. Hendricks, Agricultural Statistician

INTRODUCTION

Most agricultural statisticians who have utilized sample data from crop reporters as a basis for estimates of crop acreages have appreciated the need for a more objective method of obtaining data. A number of years ago, the field agent for South Carolina of the Bureau of Crop Estimates (now known as the Agricultural Statistics Division, Agricultural Marketing Service) made counts of the number of individual fields throughout the State in which various crops were growing. The method was crude but showed indications of promise. Field counts from trains were used in a number of States for several years. Since the average size of the fields is as important as their number, the method was extended to include an estimate of the total frontage of a given crop along the railroad right of way. This was accomplished by recording the number of telephone or telegraph poles opposite the fields planted to each crop, and was known as the "pole count."

These rather crude procedures eventually led to a more refined method of measuring changes in crop acreages that was developed by the agricultural statistician for Mississippi about 15 years ago, and has become increasingly popular as an objective method of supplementing other sources of information available to the Department of Agriculture. This method is based on the measurement, in linear units, of the frontage of cotton, corn, wheat, and other crops along a highway, the unit being taken, for convenience, as 0.02 of a mile. The measurement is performed with a "crop meter" attached to the instrument panel of an automobile and driven by a speedometer cable, as shown in figure 1.

The operation of the instrument requires very little explanation. A large dial registers the total distance driven and a number of smaller dials register the frontage measurements of various crops. The appropriate dial is put in gear by means of a push button, when the automobile

is flush with the first corner of the field, and continues to register until a release button is pushed, when the frontage is completely measured. In the course of the trip, the total frontage for each of several crops accumulates in the various dials and one can compute the ratio of the frontage of each crop to the total length of the route.

It has been found that the total acreage of a crop, in the region traversed, tends to be proportional to the ratio obtained by dividing the total frontage measured on highways, by the length of the route covered. This relationship is illustrated in figure 2, where the South Carolina cotton acreages for the years 1928-39, inclusive, have been plotted against the crop-meter ratios for the same years. In actual practice, an estimate of the acreage of a crop based on crop-meter readings is often obtained by computing the percent increase in relative frontage above the amount obtained on the same route the preceding year and equating this ratio to the percent increase in total acreage. This procedure eliminates the necessity for determining the regression of total acreage on relative frontage and eliminates some sources of variability from the data. These possible sources of variability are discussed later in the report. From a few hundred to 7,500 miles of route are covered each year in the States where the crop-meter is used and an attempt is made to keep the routes as nearly identical from year to year as possible.

The proportionality between total acreage in a region and relative frontage on highways is an interesting relationship when one considers the fact that one variable is expressed in linear units while the other is expressed in units of area. One of the first attempts to explain this relationship may be found in some notes, prepared by S. A. Jones and J. B. Shepard of the Agricultural Marketing Service, that were made available to statisticians in the crop and livestock reporting service by means of a field memorandum, issued under date of February 12, 1927. The mathematical part of the discussion is an attempt to show that the relative frontage of a given crop on highways tends to be equal to the relative area occupied by that crop in the tract of land traversed.

FIGURE 2

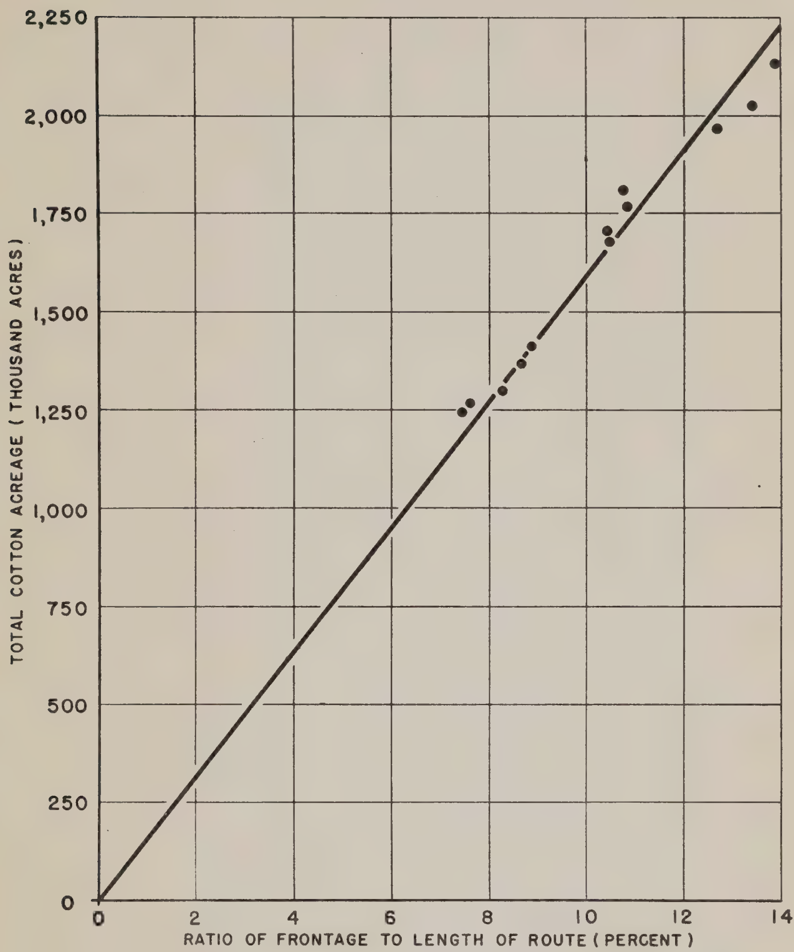

Figure 2.—Relation between South Carolina planted cotton acreage 1928-39 and the ratio of cotton frontage to length of route. The position of each dot represents the frontage ratio on the crop-meter route as compared with the total cotton acreage in the State for a given year. The straight line through the origin was fitted so that the sum of the vertical deviations of the 12 dots from the line is equal to zero.

This conclusion was based on two fundamental assumptions. The first assumption is that, on the average, the areas of individual fields tend to be equal to the squares of their frontages when areas and frontages are expressed in comparable units. This assumption appears to be reasonable provided there is no consistent tendency for either a long side, or a short side of a field to be laid off along the side of the highway. The second assumption is that the probability that a field of a given size will lie along one edge of a square tract of land and be measured, is equal to the square root of the relative area occupied by such a field in the square tract of land under consideration. The implications of this assumption can be visualized by referring to figure 3.

Figure 3.—Relation between the number of square units of area in a square mile of land and the number of such units fronting on a mile of road.

FIGURE 3

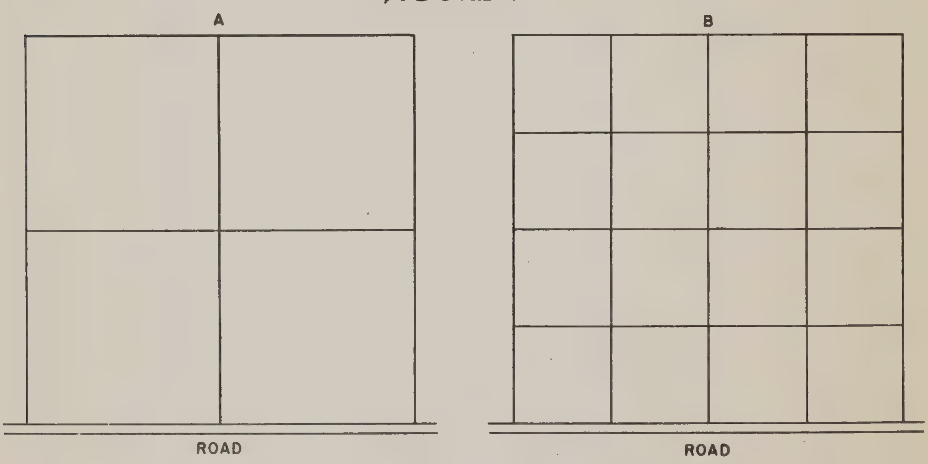

Figure 3 represents two tracts of land, each a mile square, fronting on a highway. Tract A is divided into 4 equal square units and tract B is divided into 16 equal square units. Tract A has 2 units fronting on the road while tract B has 4 units fronting on the road. If a field $1/4$ of a square mile in area can assume only one of the 4 positions shown in figure 3-A, the probability that the field will front on the road is obviously

2/4 or 1/2 which is equal to $\sqrt{1/4}$. If a field 1/16 of a square mile in area can assume only one of the 16 positions shown in figure 3-B, the probability that the field will front on the road is 4/16 or 1/4 which is equal to $\sqrt{1/16}$.

It is evident that, if the validity of the second assumption given above is to be conceded, one must be willing to postulate a restriction on the number of possible positions that a field can occupy in a given tract of land. Such a restriction might be effected by the presence of other crops. For instance, if in figure 3-A one assumes that all four quarter-sections are planted to crops, such a restriction must certainly be in operation. If one plot is a wheat field, one a corn field, one a rye field, and the fourth a hay field, there is obviously only one chance in two that the wheat field will front on the road. If the entire tract is not used, however, such a restriction will not take place. This being the case, it appeared desirable to learn the extent to which the simple hypothesis used to explain relationships, such as the one shown in figure 2, is justified in fact.

Nature of the Present Study

In testing the validity of any hypothesis, one is usually concerned with deducing the necessary consequences of that hypothesis, and noting the extent to which observed phenomena are in agreement with what is expected under the hypothesis. It may be stated at the beginning that the results to be presented in this publication confirm the essential features of the assumptions under consideration. The necessary data were obtained from aerial survey photographs, made in 1937, for the following counties on which crop identification had been made:

Washington County, Indiana	Gentry County, Missouri
Wayne County, Indiana	Harlan County, Nebraska
Jefferson County, Iowa	Dane County, Wisconsin
McLeod County, Minnesota	Rock County, Wisconsin

The study was made on a county basis and the work was limited to a consideration of only three crops—corn, wheat, and alfalfa. Corn and wheat are major crops in practically all the eight counties, while alfalfa is a comparatively minor crop. Several highways through each county were identified on road maps and traced on the photographs. An attempt was made to obtain a number of approximately parallel routes going north and south in each county, and a number of approximately parallel routes going east and west in each county. This was accomplished fairly successfully in all counties except Dane County, where highways tend to radiate from Madison, the centrally located State capital of Wisconsin. However, a sufficiently large number of routes was available so that Dane County was covered as thoroughly as the others.

The various phases of the study herein reported deal with relationships that one would expect to find if the fundamental hypothesis under test is valid. Five such relationships were tested and may be listed briefly as follows:

1. The relation between frontages and areas of individual fields.
2. The relation between total crop acreages in each county and the corresponding acreages computed from frontage measurements.
3. The relation between the observed number of fields per mile of route and the number expected under the hypothesis.
4. The relation between the average size of fields in the county and the average size of those fields fronting on the routes.
5. The relation between the observed variability of relative frontages and the variability expected under the hypothesis.

These relationships are discussed in detail in the succeeding sections of this report.

Relation Between Frontages and Areas of Individual Fields

The first assumption to be tested was the one stating that, on the average, the areas of individual fields tend to be equal to the squares of their frontages. From the point of view of the subsequent discussion it is more convenient to restate this assumption by saying that frontages of individual fields tend to be equal to the square roots of their areas.

If there are $n_1, n_2, —, n_p$ fields with areas $A_1, A_2, —, A_p$ and frontages $F_1, F_2, —, F_p$, respectively, where areas are expressed in square miles and frontages in miles, a proportionality between the frontage of an individual field and the square root of its area can be represented by the equation,

$$F_i = c_1 \sqrt{A_i} \qquad (1)$$

The hypothesis under test states that c_1 is equal to unity.

To test the relation indicated by equation (1), a route was chosen at random in each county and the area and frontage of each of 100 successive fields on this route, growing either corn, wheat, or alfalfa, were measured. It was found that, for each county, the areas of individual fields tended to be proportional to the frontages rather than to the squares of the frontages. In other words, it appeared that on any given segment of road there is a tendency for depths and frontages of individual fields to vary independently about their average values. The factor of proportionality, however, was found to depend on the average size of the fields for each county so that the average areas were proportional to the squares of the average frontages. The results of this phase of the study are summarized in table 1 and presented graphically in figure 4.

It is evident that the data in the last column of table 1 are not correlated with the average areas of the fields. The average of these values, 201.173, provides an estimate of c_1 in equation (1). If areas and frontages are expressed in comparable units, this constant has the value 0.96389. Its standard error is 0.01331. Thus, the value of c_1 appears

FIGURE 4

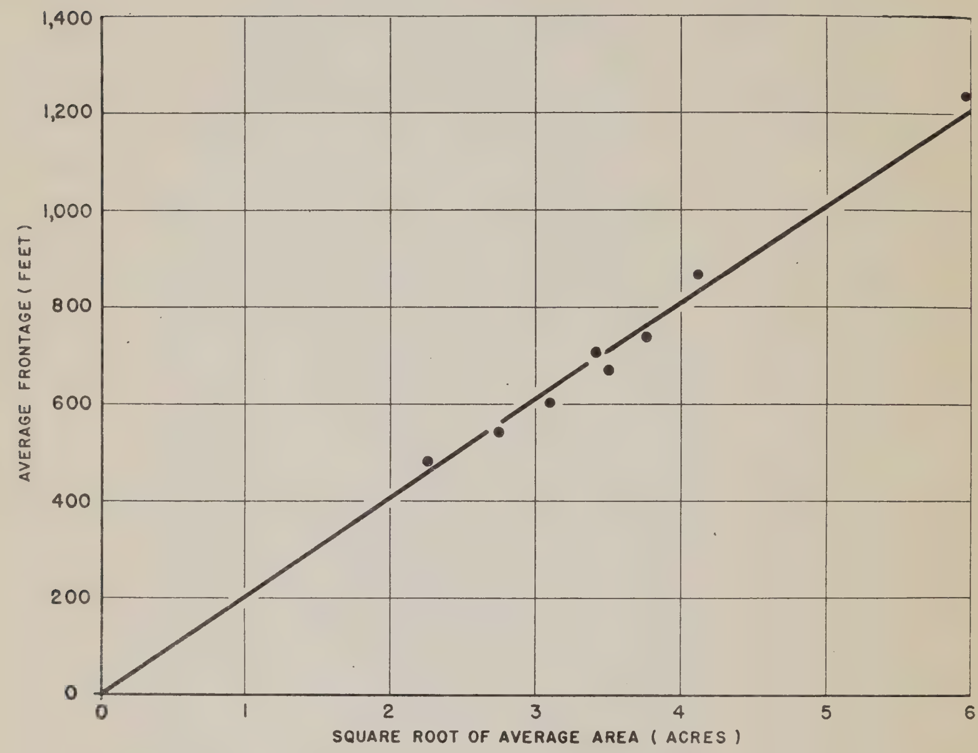

Figure 4.—Relation between average frontage and average area of individual fields on a route. The position of each dot represents the square root of the average area of 100 individual fields on a random route in a county as compared with the average frontage of those fields. Each dot represents data for a different county. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

Table 1.—Relation Between Area and Frontage for 100 Individual Fields on a Random Route Through Each County

COUNTY	Av. area	Av. frontage	Av. frontage / Av. area	Av. frontage / Square root of Av. area
	Acres	Feet		
Dane, Wis.	5.127	476.30	92.900	210.35
McLeod, Minn.	7.559	539.59	71.384	196.26
Wash., Ind.	9.579	602.10	62.856	194.54
Gentry, Mo.	11.699	701.23	59.939	205.01
Rock, Wis.	12.244	667.11	54.485	190.65
Wayne, Ind.	14.172	734.70	51.842	195.17
Jefferson, Iowa	16.959	865.70	51.047	210.22
Harlan, Neb.	35.748	1238.75	34.652	207.18
Average				201.173

to be significantly different from unity. However, in the absence of any logical explanation of such a difference, little importance can be attached to it. The difference might have arisen as a sampling fluctuation, though it is somewhat larger than one might expect. The observed value was used in all the subsequent computations involving c_1 to avoid the possibility of introducing an error.

Observed Crop Acreage and Acreages Computed from Frontage Measurements

The fundamental data required for studying the relation between the crop acreages in a county and the corresponding frontage measurements on highways are given in table 2.

The geographic areas of the counties given in table 2 are somewhat empirical. In some cases the county boundaries could not be clearly distinguished on the aerial photographs, and in some cases small portions of a county had to be neglected because crop identification was incomplete. In all cases, however, the areas given are the measured areas to which the field counts and crop measurements apply. The data in the

Table 2.—Data Relating to Field Counts and Crop Measurements for Each County

County	Geographic area of county*	Total Length of all routes in county	Crop	No. fields in county	Area of all fields in county	No. fields on all routes in county	Area of fields on all routes in county	Crop frontage on all routes in county
	Sq. miles	miles		number	acres	number	acres	feet
Dane, Wis.	1219.49	299.53	Corn	16,853	138,415	867	7,580.7	492,336
			Wheat	674	2,799	17	59.0	6,864
			Alfalfa	8,694	48,354	489	3,005.2	255,578
McLeod, Minn.	500.77	149.53	Corn	5,003	57,832	308	4,120.8	215,584
			Wheat	2,327	20,837	117	1,092.3	72,239
			Alfalfa	3,507	15,568	238	1,217.7	130,383
Washington, Ind.	500.12	183.60	Corn	5,293	36,609	397	3,538.4	247,835
			Wheat	1,736	17,266	153	1,689.7	95,475
			Alfalfa	471	2,986	65	412.4	39,420
Gentry, Mo.	489.35	217.98	Corn	4,491	45,047	338	3,859.5	229,213
			Wheat	2,308	29,005	190	2,395.7	134,951
			Alfalfa	504	2,968	37	236.9	22,144
Rock, Wis.	716.94	224.13	Corn	9,264	107,439	685	8,346.5	431,787
			Wheat	684	4,121	46	259.1	23,760
			Alfalfa	1,407	9,120	116	705.5	55,930
Wayne, Ind.	374.59	149.72	Corn	4,776	63,636	401	5,655.7	305,479
			Wheat	2,268	36,825	226	3,466.7	178,926
			Alfalfa	1,388	10,740	134	1,051.0	81,613
Jefferson, Iowa	435.43	173.52	Corn	4,489	62,149	492	8,101.0	386,095
			Wheat	864	9,512	89	1,065.7	62,520
			Alfalfa	191	1,239	17	105.9	8,350
Harlan, Nebr.	572.19	158.047	Corn	2,732	71,678	311	9,282.8	304,354
			Wheat	2,216	83,222	341	15,050.6	421,107
			Alfalfa	395	3,375	35	270.9	19,672

*Entire area included within county boundaries

last three columns of the table refer to counts and measurements made on both sides of the routes. The numbers of fields, areas, and frontages appeared to be approximately the same on each side of every route. Thus, any formula requiring counts or measurements on only one side of a route can be applied if the data in the last three columns of the table are divided by 2.

The nature of the mathematical relationship between crop acreages and frontage measurements may now be discussed from the point of view of the hypothesis under test. If in a square mile of land, there are n_1, n_2, —, n_p fields with areas A_1, A_2, —, A_p, where areas are expressed in square miles; and the probability that a field of area A_i will lie on one side of a mile of route is proportional to $\sqrt{A_i}$, the number of fields of that size on one side of the route may be represented by the equation,

$$n_1' = c_2 \, n_i \sqrt{A_i} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where n_i' is the number of fields of a given size on one side of a mile of route and n_i is the number of such fields per square mile of area. The total area of all fields in a square mile is equal to the sum of the areas of the individual fields, or $S(n_i \, A_i)$. The total frontage of those fields that fall on one side of a mile of route is $S(n_i' \, F_i)$. By making use of equations (1) and (2), this expression can be written in the form $S(c_2 \, n_i \sqrt{A_i} \; c_1 \sqrt{A_i})$ or $c_1 \, c_2 \, S(n_i \; A_i)$. The ratio of the total area of the fields in the square mile to the total frontage of those that fall on one side of a mile of route is, therefore, equal to $1/(c_1 c_2)$.

It is important to notice that equation (1) is concerned with the relationship between frontage and area of fields which actually lie on the route. Equation (2) gives the relation between the number of fields in a square mile of area and the number that lie on a mile of route. Combining these two equations establishes a relationship between the frontage of the fields which actually lie on a mile of route and the area of all fields in a square mile, all of which will not be on the route.

As a preliminary step in the analysis of the data from this point of view, the relative frontage of each of the three crops was computed for each county by dividing the total frontage by the length of the route. The relative frontage was multiplied by the area of the county to obtain an estimate of the crop acreage in the county under the assumption of equality of relative frontage and relative acreage. The results are summarized in table 3.

Table 3.—Observed Crop Acreages and Acreages Computed by Assuming Equality Between Relative Frontages and Relative Acreages

COUNTY	CORN		WHEAT		ALFALFA	
	Observed	Computed	Observed	Computed	Observed	Computed
	acres	acres	acres	acres	acres	acres
Dane, Wis.	138,415	121,483	2,799	1,694	48,354	63,063
McLeod, Minn.	57,832	43,756	20,837	14,662	15,568	26,463
Washington, Ind.	36,609	40,915	17,266	15,762	2,986	6,508
Gentry, Mo.	45,047	31,186	29,005	18,361	2,968	3,013
Rock, Wis.	107,439	83,708	4,121	4,606	9,120	10,843
Wayne, Ind.	63,636	46,321	36,825	27,131	10,740	12,375
Jeff., Iowa	62,149	58,719	9,512	9,508	1,239	1,270
Harlan, Neb.	71,678	66,781	83,222	92,398	3,375	4,316
Total	582,805	492,869	203,587	184,122	94,350	127,851

The total observed acreage in the three crops is 880,742, while the computed acreage is only 804,842. This indicates that the value of c_2 in equation (2) is not equal to unity. Furthermore, it is apparent that the corn and wheat acreages were consistently underestimated whereas the alfalfa acreages were consistently overestimated. This indicates that the value of c_2 tends to vary from crop to crop.

It was conceivable that the value of c might vary from county to county as well as from crop to crop. The crop-to-crop variation can be explained by a tendency for certain crops to be planted near highways. A county-to-county variation could be explained by differences in the topography of the counties, or differences in the nature of the highways that might easily affect the probability of encountering crops in general. Some highways tend to pass through nonagricultural regions; others tend to limit themselves to agricultural regions.

In order to obtain some information on these points, the ratio of the observed acreage to the corresponding computed acreage for each crop was obtained for each of the 8 counties. The resulting 24 ratios were investigated by analysis of variance. One need reflect only a moment to conclude that, if there is a county-to-county variation and a crop-to-crop variation, the composite effect of these sources of variability on the ratio for a given crop in a given county is multiplicative rather than additive. For this reason, the analysis of variance was performed with the logarithms of the ratios rather than with the ratios themselves. The results of the analysis are summarized in table 4.

Table 4.—Analysis of Variance of Log $\frac{\text{Relative acreage}}{\text{Relative frontage}}$

Source of variability	Degrees of freedom	Sum of squares	Mean square	F
Between crops	2	0.20409	0.102045	14.354
Between counties	7	.10650	.015214	2.140
Error	14	.09953	.007109	
Total	23	0.41012	0.017831	

The variance between crops is highly significant but the variance between counties is not significant. These results indicated that the tendency for certain crops to be planted on highways was fairly consistent from county to county. The county-to-county variation was not sufficiently large for all crops, however, to make it worth while to

attempt an adjustment on the computed acreages in table 3 that took this factor into consideration. An adjustment for the crop bias is all that appeared to be required at this stage of the analysis.

The ratio of total observed acreage to total computed acreage was obtained for each crop. These ratios have the values

Corn	1.18247
Wheat	1.10572
Alfalfa	0.73797

Figures 5, 6, and 7 show the effects of this crop bias in graphical form. When the data in table 3 were adjusted for the bias, the results shown in table 5 were obtained.

Table 5.—Observed Crop Acreages and Computed Acreages Adjusted for Crop Bias

County	Corn		Wheat		Alfalfa	
	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres
Dane, Wis.	138,415	143,650	2,799	1,873	48,354	46,539
McLeod, Minn.	57,832	51,740	20,837	16,212	15,568	19,529
Washington, Ind.	36,609	48,381	17,266	17,428	2,986	4,803
Gentry, Mo.	45,047	36,877	29,005	20,302	2,968	2,224
Rock, Wis.	107,439	98,982	4,121	5,093	9,120	8,002
Wayne, Ind.	63,636	54,773	36,825	29,999	10,740	9,132
Jefferson, Iowa	62,149	69,433	9,512	10,513	1,239	937
Harlan, Nebraska	71,678	78,967	83,222	102,166	3,375	3,185
Total	582,805	582,803	203,587	203,586	94,350	94,351

FIGURE 5

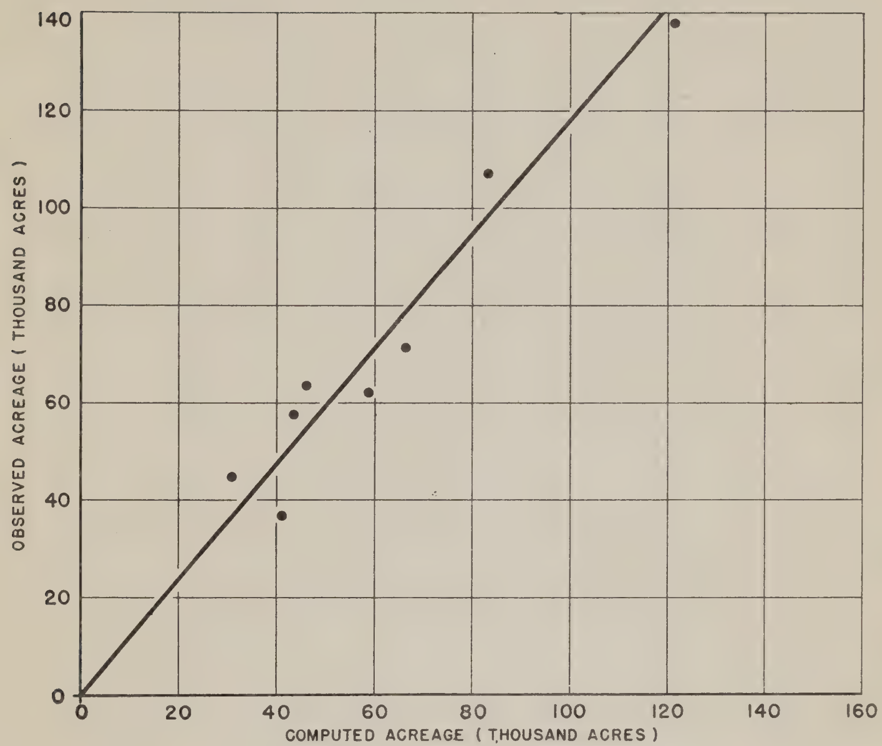

Figure 5.—Total corn acreage in each county and acreage computed by assuming equality between relative acreage and relative frontage. The position of each dot represents the computed acreage in a county as compared with the observed acreage. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

FIGURE 6

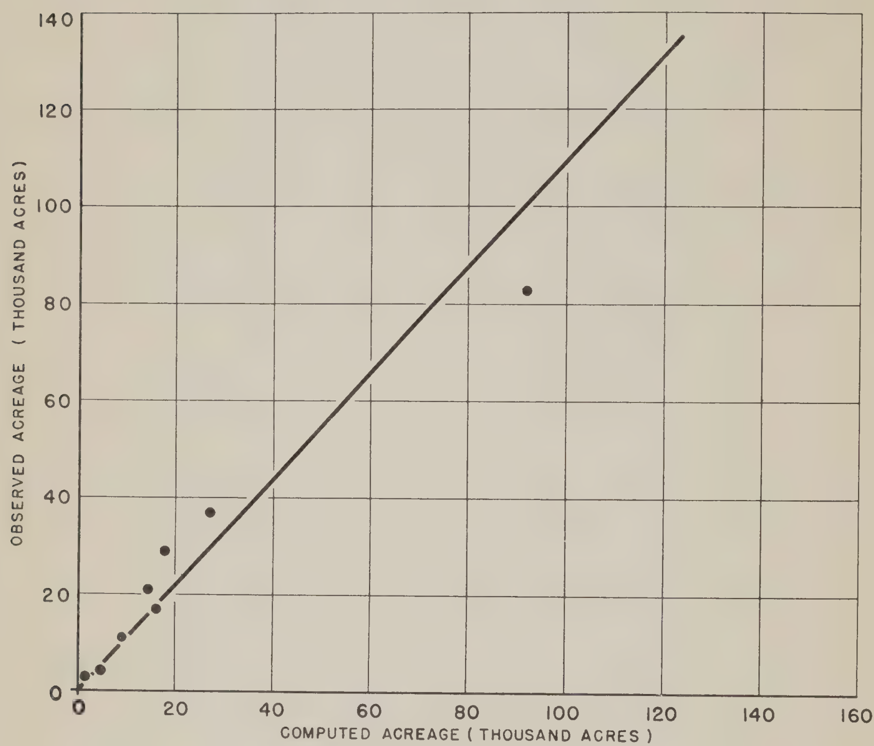

Figure 6.—Total wheat acreage in each county and acreage computed by assuming equality between relative acreage and relative frontage. The position of each dot represents the computed acreage in a county as compared with the observed acreage. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

FIGURE 7

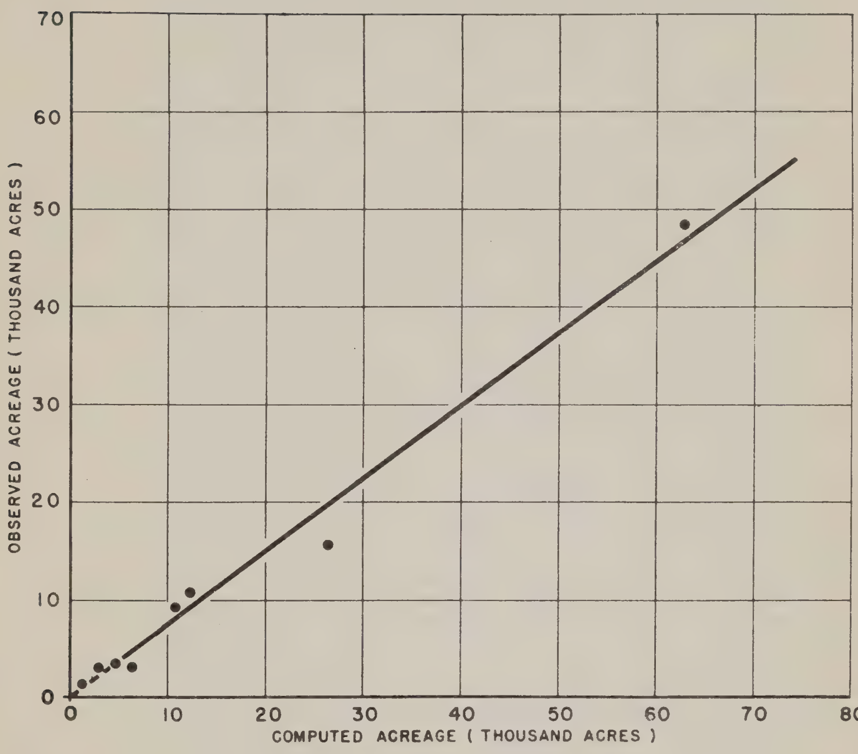

Figure 7.—Total alfalfa acreage in each county and acreage computed by assuming equality between relative acreage and relative frontage. The position of each dot represents the computed acreage in a county as compared with the observed acreage. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

Number and Average Size of Fields per Mile of Route in Relation to Distribution of Field Sizes in County

The results presented so far show that the relative acreage of a given crop in a county is proportional to the relative frontage of that crop on highways. The analysis would not be complete without a study of the relationship between the number of fields per square mile of area in a county and the number of fields per mile of route in that county for each crop, together with a consideration of the average size of the fields on the routes as compared with the average for the entire county. According to equation (2), the number of fields of a given size which can be expected on a mile of route is equal to the product $c_2 \; n_i \sqrt{A_i}$, in which A_i is the area of a single field, expressed in square miles, and n_i is the number of fields of that size per square mile of area in the county.

The value of c_2 for each crop can be estimated from the three crop bias ratios given above. According to the theory, each of these ratios is the value of the quantity $\frac{1}{c_1 c_2}$ for the corresponding crop. If the value of c_1 is taken to be 0.96389, the values of c_2 computed from the above ratios are

Corn	0.87737
Wheat	0.93827
Alfalfa	1.40584

It would appear to be a simple matter to apply equation (2) to the observed data. If a county contains n fields of a given crop per square mile of area and n_i represents the number of fields of any given area A_i, the number of fields in that crop per mile of route should be equal to $c_2 S (n_i \sqrt{A_i})$. However, as a practical matter, the computation of the value of this expression would be burdensome. The procedure followed in the present study does not differ in principle from that indicated above, but can be applied with much less labor. Briefly,

the method consisted of deriving a mathematical equation to represent the frequency distribution of individual fields for each crop in a county and replacing the sum in the expression $c_2 \; S(n_i \sqrt{A_i})$ by a definite integral.

The function that seemed appropriate to represent this frequency distribution was a Pearsonian Type III Curve of the form,

$$df_A = \frac{a^{b+1}}{\Gamma(b+1)} e^{-aA} A^b \, dA \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

in which a and b are parameters whose numerical values in any given instance depend upon the average size of the fields and the coefficient of variation* of the individual fields.

The average size of the fields for each crop in each county was computed from the data in table 2 but the coefficient of variation was computed indirectly. According to the theory, the probability that a field lies on a mile of route is proportional to the square root of its area. If this is true, the proportion of large fields found on the routes in a county will be greater than the proportion in the county as a whole. If the frequency distribution of the fields in the county is given by equation (3) it may be shown that the coefficient of variation is given by the formula,

$$v^2 \doteq \frac{2(\bar{A}_r - \bar{A})}{\bar{A}} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

in which v represents the coefficient of variation, $\bar{A}_r$ represents the average area of the fields on the routes, and $\bar{A}$ represents the average area of the fields in the county. The derivation of this formula is given in the Appendix to this report for the benefit of those interested in the mathematical aspects of the problem.

It should be noted that the average area of the fields on the routes must necessarily be greater than the average area of all the fields in the county if the fundamental theory underlying the use of the crop meter

* In this report the "coefficient of variation" is defined as the ratio of the standard deviation to the arithmetic mean.

is valid. The author has frequently heard statements to the effect that an efficient use of the crop meter presupposes that the fields on the routes are a "good" or a "fair" or a "representative" sample of the fields in the region. Such statements are inconsistent with the theory under discussion. One cannot, at one stage of an argument, assume that large fields have a greater probability of being encountered on a route than small fields and, at another stage, that the fields on the route are a representative sample of the fields in the region traversed by the route.

An examination of the data indicated that the value of v tended to be constant from crop to crop and from county to county. Equation (4) shows that, if v is constant, the relative bias in the average area of the fields on the routes is also constant. In view of these considerations, the data for corn, wheat, and alfalfa were combined and the relative bias in the average area of the fields on the routes was computed from the combined data for each county. The results are summarized in table 6 and are presented graphically in figure 8.

Table 6.—Average Area of Corn, Wheat, and Alfalfa Fields in Each County and on all Routes in Each County

County	Average area of fields in county	Average area fields on routes	Bias	
	acres	acres	acres	relative
Dane, Wis.	7.230	7.753	0.523	0.07234
McLeod, Minn.	8.696	9.700	1.004	.11546
Washington, Ind.	7.581	9.172	1.591	.20987
Gentry, Mo.	10.546	11.490	.944	.08951
Rock, Wis.	10.628	10.993	.365	.03434
Wayne, Ind.	13.188	13.368	.180	.01365
Jefferson, Iowa	13.149	15.506	2.357	.17925
Harlan, Nebraska	29.623	35.814	6.191	.20899
Average				0.11543

FIGURE 8

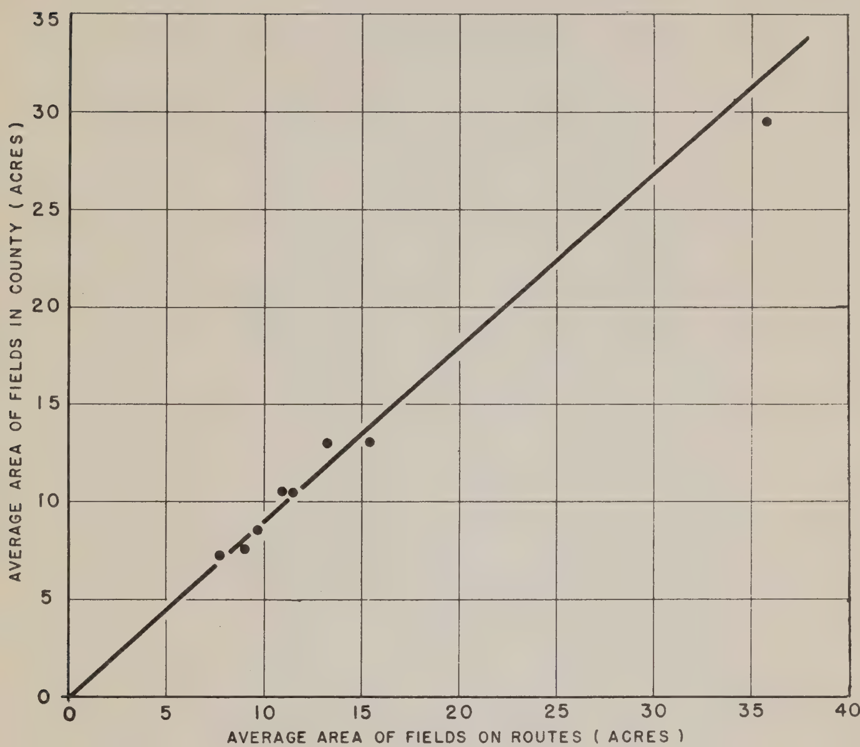

Figure 8.—Average area of fields in each county and average area of fields on routes in each county. The position of each dot represents the average area of the fields on the routes in a county as compared with the average area of all fields in the county. Only fields growing either corn, wheat, or alfalfa were considered. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

The average relative bias shown in table 6 is 0.11543. The coefficient of variation of individual fields, therefore, is equal to the square root of 0.23086 or 0.48048. This value of v, together with the average area of the fields for each crop, provides all the information required tc compute the values of a and b for each crop by counties. Thus, the frequency distribution of the size of field is established for each crop in each county. By means of these distributions it may be shown that the number of fields that can be expected on a mile of route is given by the formula,

$$n_r = 0.97163\ c_2\ n\sqrt{\bar{A}} \quad . . . . \quad (5)$$

in which n_r is the number of fields of a given crop per mile of route, n is the number of such fields per square mile of area in the county, and $\bar{A}$ is the average area of all such fields in the county, expressed in square miles. The numerical values of c_2 have been given previously in this report. The derivation of equation (5) may be found in the Appendix.

The observed numbers of fields per mile of route and the corresponding numbers computed by means of equation (5) are shown in table 7. The numbers given refer to counts on only one side of a route.

Table 7.—Observed and Computed Numbers of Fields Per Mile of Route

County	Corn		Wheat		Alfalfa		Total	
	Observed number	Computed number	Observed number	Computed number	Observed number	Computed number	Observed number	Computed number
Dane, Wis.	1.447	1.335	0.028	0.041	0.816	0.908	2.291	2.284
McLeod, Minn.	1.030	1.145	.391	.501	.796	.797	2.217	2.443
Washington, Ind.	1.081	.938	.417	.394	.177	.128	1.675	1.460
Gentry, Mo.	.775	.979	.436	.602	.085	.135	1.296	1.716
Rock, Wis.	1.528	1.482	.103	.084	.259	.270	1.890	1.836
Wayne, Ind.	1.339	1.568	.755	.880	.448	.557	2.542	3.005
Jefferson, Iowa	1.418	1.292	.256	.237	.049	.060	1.723	1.589
Harlan, Nebraska	.984	.824	1.079	.855	.111	.109	2.174	1.788

The observed numbers of fields for each crop are plotted against the corresponding computed values in figure 9. It is quite apparent that the agreement is satisfactory. This indicates that the differences in the values of c_2, from crop to crop, are due to differences in the probability of encountering certain crops on highways and are not spurious effects caused by such factors as differences in field shape.

Observed and Expected Standard Error of a Relative Frontage

The results presented thus far show good agreement between observation and theory. The hypothesis may be subjected to a more stringent test than any of those previously considered by deducing the expression for the standard error of a relative frontage from the theory and comparing the observed variability with the variability which would be expected if the hypothesis under consideration were completely valid in all respects.

To deduce the formula for the standard error of a relative frontage from theoretical considerations, consider a tract of land containing to the square mile n_1, n_2, —, n_p fields whose individual areas, expressed in square miles, are A_1, A_2, —, A_p. Consider the n_1 fields of area A_1.

Each square mile may be divided into $1/A_1$ equal square spaces, each of area A_1. When one of these spaces is taken at random, the probability that it will contain one of the n_1 fields is $n_1 \div 1/A_1$ or $n_1 A_1$. However, if there is a tendency for the crop to be located on the road or away from the road, the probability that a space will contain a field is $c_2 n_1 A_1$. By taking a random route of length k miles one is, in effect, taking a sample of $\frac{k}{\sqrt{A}}$, such spaces, since this is the number of such spaces that can be placed side by side on a route k miles long. The expected number of fields on k miles of route is the

FIGURE 9

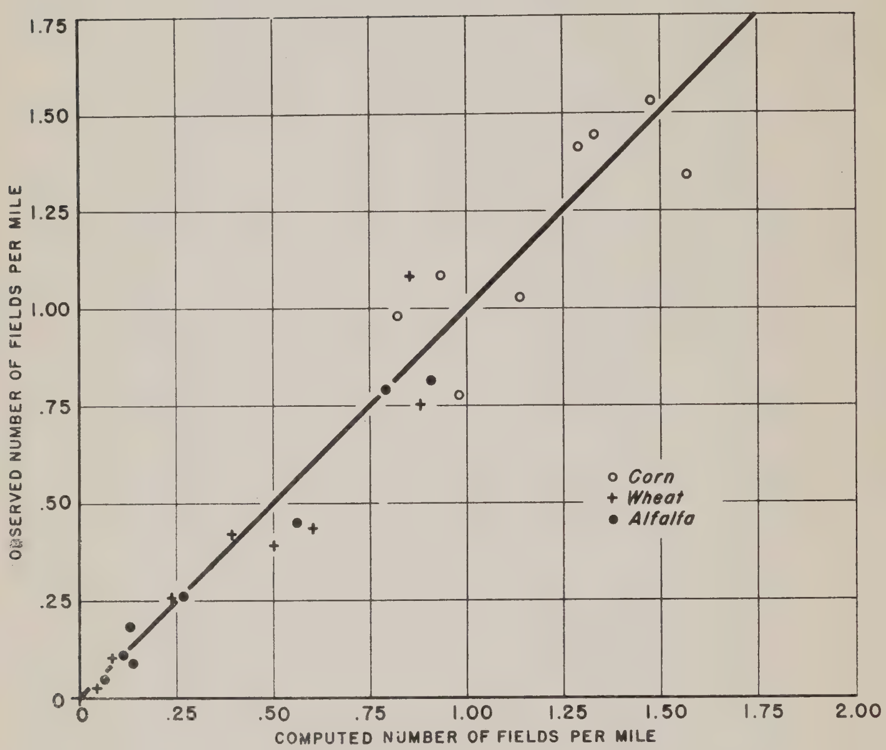

Figure 9.—Observed and computed numbers of fields per mile of route in each county. The position of each dot represents the computed number of fields growing a given crop that was expected on a mile of route in a county as compared with the average number actually found. The straight line through the origin is the expected line of best fit if no bias is present. It was not fitted to the data.

product of $c_2n_1A_1$ and $k/\sqrt{A_1}$ or $kc_2n_1\sqrt{A_1}$ and the variance of the number of fields on k miles of route is $kc_2n_1\sqrt{A_1}(1-c_2n_1A_1)$. Since the number of fields having exactly the area A_1 may be considered small in relation to the total number of fields, the quantity in parentheses is nearly equal to unity and the variance may be taken simply as $kc_2n_1\sqrt{A_1}$. The variance of the number of fields on k miles of route, therefore, is equal to the expected number of fields.

The expected frontage of each of these fields is $c_1\sqrt{A_1}$ but the frontages of individual fields of a given area are subject to a certain amount of variability. Table 8 gives the variance of the 100 fields, previously considered, on the random route for each county investigated and shows that the variance is proportional to the average area of the fields. The data are presented graphically in figure 10.

Table 8.—Variance of Frontage of 100 Individual Fields in Each County

County	Variance of frontages FEET	Average area of fields ACRES	$\frac{\text{Variance}}{\text{Average area}}$
Dane, Wis.	117,162	5.127	22,852
McLeod, Minn.	137,626	7.559	18,207
Washington, Ind.	193,756	9.579	20,227
Gentry, Mo.	225,008	11.699	19,233
Rock, Wis.	272,018	12.244	22,216
Wayne, Ind.	147,332	14.172	10,396
Jefferson, Iowa	186,152	16.959	10,977
Harlan, Nebraska	757,659	35.748	21,194
Average			18,162.8

FIGURE 10

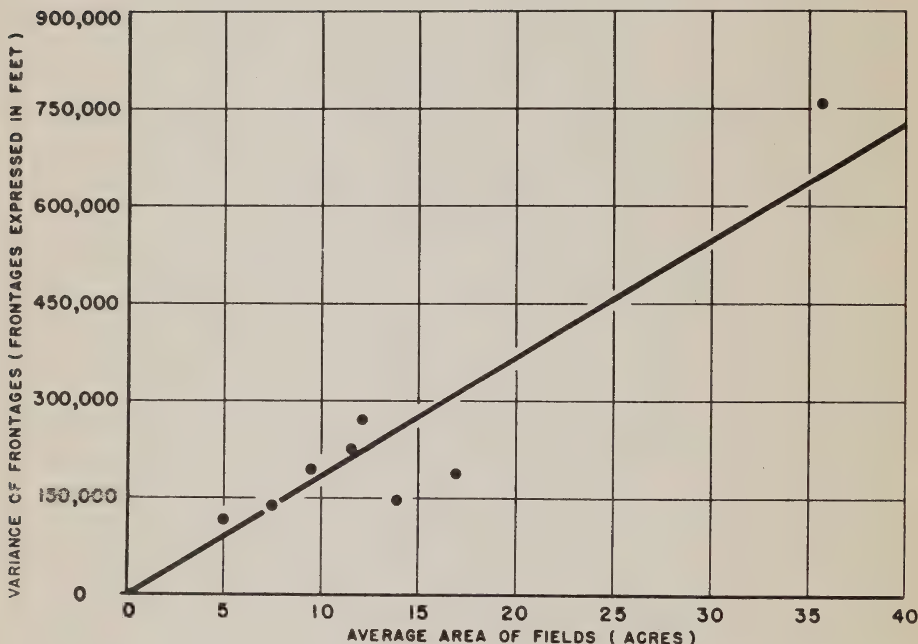

Figure 10.—Relation between variance of frontages and average area of individual fields. The position of each dot represents the variance of the frontages of 100 individual fields on a random route in a county as compared with the average area of those fields. Each dot represents data for a different county. The slope of the straight line through the origin was computed by taking the unweighted average of the ratios of the 8 variances to the corresponding 8 average field areas.

When frontages are expressed in miles and areas in square miles, the average factor of proportionality in table 8 is equal to 0.41696. This constant is denoted by c_3 in the following discussion.

The variance of the total frontage on k miles of route consists of two components. The first is due to the effect of variation in the number of fields and is equal to $(c_1\sqrt{A_1})^2 (k\, c_2\, n_1\sqrt{A_1})$ or $k\, c_1^2\, c_2 n_1 A_1^{3/2}$. The second is due to the variability of the frontages of fields of the same area and is approximately equal to $(kc_2 n_1 \sqrt{A_1})(c_3 A_1)$ or $kc_2c_3n_1A_1^{3/2}$. The variance of the total frontage of fields of area A_1 is equal to the sum of these two components or $k(c_1^2 + c_3)c_2n_1A_1^{3/2}$.

The variance of the total frontage contributed by all the fields is obviously equal to $k\,(c_1^2 + c_3)\,c_2\, S\,(n_i\, A_i^{3/2})$, an expression which can be evaluated by integration. The variance of the relative frontage is obtained by dividing the result by k^2. When these operations are performed, the formula for computing the variance of a relative frontage, based on k miles of route, can be written in the form,

$$s^2 = \frac{1.0838(c_1^2 + c_3)c_2 n\bar{A}^{3/2}}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

The number of fields of a given crop per square mile of area is represented by n and the average area of those fields, expressed in square miles, is represented by $\bar{A}$.

Equation (6) was tested by comparing results given by the formula with the discrepancy between the observed and expected relative frontages for each crop in each county. If the relative area of a crop acreage in a given county is written in the form $n\bar{A}$, the expected relative frontage of the crop on routes is equal to $c_1\, c_2\, n\, \bar{A}$. Values of this quantity can be computed for each crop in each county and compared

with the corresponding observed values. The sum of the squares of the differences each divided by the corresponding variance as computed from equation (6), should be distributed as chi square if the theory is valid.

Preliminary work on this problem showed that the use of only three different values of c_2, correcting for crop bias, were not sufficient to reduce the variability of the relative frontages to an amount comparable with the variances computed by equation (6). Therefore eight additional factors correcting for county bias were computed and used in the analysis. The proper value of k to be used in equation (6) is twice the total miles driven in each county, since the relative frontages observed are based on observations taken on both sides of every route.

If the computed acreages in table 3 are multiplied by $\frac{880742,}{804842}$ the grand total of the computed acres in the table will be brought into agreement with the total of the observed acres, though the various crop and county totals will not be equal. Therefore, the adjusted computed acreage for each crop in each county must be multiplied by an additional factor of the form $t_i t_j$ in which the t_i represent the three crop bias factors and the t_j represent the eight county bias factors.

Although it is theoretically possible to compute the values of these constants from the data, such a computation would involve the solution of equations of considerable complexity. Since the t_i and t_j do not differ greatly from unity, a serviceable approximation is obtained by writing $t_i t_j = (1 + x_i)(1 + y_j)$ which is approximately equal to $1 + x_i + y_j$, when the x_i and y_j are small. The numerical values of the x_i and y_j were computed from the data so that when factors of the form $1 + x_i + y_j$ were used as multipliers of the computed acres, as adjusted above, the crop and county totals were equal to the corresponding observed totals. A comparison of the final adjusted computed acreages with the corresponding observed acreages is given in table 9.

Table 9.—Observed Crop Acreages and Computed Acreages Adjusted for Crop and County Bias

County	Corn		Wheat		Alfalfa		Total	
	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres
Dane, Wis.	138,415	143,137	2,799	1,954	48,354	44,475	189,568	189,566
McLeod, Minn.	57,832	55,218	20,837	18,139	15,568	20,879	94,237	94,236
Washington, Ind.	36,609	39,067	17,266	14,658	2,986	3,136	56,861	56,861
Gentry, Mo.	45,047	46,815	29,005	27,107	2,968	3,098	77,020	77,020
Rock, Wis.	107,439	106,304	4,121	5,734	9,120	8,642	120,680	120,680
Wayne, Ind.	63,636	63,538	36,825	36,542	10,740	11,121	111,201	111,201
Jefferson, Iowa	62,149	62,301	9,512	9,852	1,239	747	72,900	72,900
Harlan, Nebraska	71,678	66,421	83,222	89,603	3,375	2,251	158,275	158,275
Total	582,805	582,801	203,587	203,589	94,350	94,349	880,742	880,739

A comparison of the observed and expected relative frontages, after adjusting for crop and county bias, is given in table 10.

Table 10.—Comparison of Observed and Expected Relative Frontages

County	Corn			Wheat			Alfalfa		
	Relative frontage observed	Relative frontage expected	Diff. / S.E.	Relative frontage observed	Relative frontage expected	Diff. / S.E.	Relative frontage observed	Relative expected frontage	Diff. / S.E.
Dane, Wis.	0.155653	0.150518	+0.782	0.002170	0.003109	−1.182	0.080801	0.087848	−1.549
McLeod, Minn.	.136528	.142990	−.655	.045749	.052052	−1.124	.082571	.061566	+4.123
Washington, Ind.	.127828	.119786	+1.122	.049244	.058006	−1.605	.020332	.019360	+.344
Gentry, Mo.	.099577	.095816	+.583	.058627	.062731	−.743	.009620	.009217	+.228
Rock, Wis.	.182434	.184381	−.212	.010039	.007214	+1.839	.023631	.024938	−.449
Wayne, Ind.	.193214	.193514	−.032	.113170	.114046	−.093	.051619	.049851	+.336
Jeff., Iowa	.210707	.210193	+.045	.034119	.032941	+.272	.004557	.007559	−1.647
Harlan, Neb.	.182360	.196794	−1.045	.252314	.234335	+1.090	.011787	.017670	−1.881

The sum of the squares of ratios of the differences to their standard errors should be equal to a value of chi square corresponding to 14 degrees of freedom. For the given data, chi square is equal to 40.36 which is excessively large for 14 degrees of freedom. It should be noted that the discrepancy between observed and expected frontage of alfalfa in McLeod county alone contributes 17 units to this value. All of the other contributions to the value of chi square appear to be of the expected order of magnitude.

It is tempting to explain the one highly aberrant observation in the table as an accidental occurrence caused by nothing more than fluctuations under random sampling. However, it is highly probable that the crop and county bias factors used in the above analysis are not adequate for determining the true bias in the relative frontage for a given crop in a given county. It seems reasonable to suppose that a crop bias need not necessarily be constant from county to county or that a county bias need not necessarily be constant for all crops. The conclusion to be drawn from the above results is that equation (6) provides a valid estimate of the variance of a relative frontage when sources of bias are removed. In actual practice, crop meter routes are chosen in such a way that bias in the results is constant from year to year, and under such conditions equation (6) is likely to yield satisfactory results.

Summary and Conclusions

The results of the study herein reported show that the relative frontage of a crop on highways in a county is proportional to the relative acreage of that crop in the county. The factor of proportionality varies from crop to crop and from county to county because of a tendency for certain crops to be planted near highways and because of differences in the topography and utilization of the land traversed by the routes. The factors of proportionality for various crops and localities do not differ greatly from unity. That fact lends considerable support to the hypothesis that, on the average, areas of individual fields tend to be equal to the squares of their respective frontages and that the chance of encountering a field on a mile of route tends to be equal to the square

root of its area, the area being expressed in square miles. Such discrepancies as were observed can be explained by sources of bias, like those noted above, which have been recognized for some time by users of the crop meter. A detailed analysis of the data confirmed this point of view.

In the practical use of the crop meter, the effects of some of these sources of bias can be eliminated by using the year-to-year change in relative frontage on identical routes as a measure of change in relative acreage. This practice is already being followed by most users of the crop meter. The effects of variability in the type of farming area traversed can be eliminated by proper stratification of the area under consideration.

The success of the crop meter depends largely upon the postulate that the region traversed be fairly homogeneous with respect to distribution of crops and topography of the land. Whenever it is possible to distinguish different types of farming regions within a county or State, and the boundaries of such regions can be defined, it seems desirable to obtain relative frontage measurements separately for each type of area and to apply them separately in computing the crop acreages even though identical routes are used from year to year. The same result could be achieved by laying out the routes in such a manner that the distance driven through each type of region would be proportional to the geographic area of the region involved. This would lead to a properly weighted average relative frontage that could be applied to the county or State as a whole.

It also appears to be important to continue the present practice of rigorously defining what constitutes a crop frontage. Such a definition should be based on considerations involving ease of application on the part of the operator engaged in making the measurements rather than on theoretical grounds. Any sources of bias introduced by the definition of a crop frontage should be constant and so long as other known con-

stant sources of bias are present no additional difficulties will be introduced. From the point of view of mathematical theory, the nature of the definition is not important. The important consideration is that the definition be followed consistently and the one easiest to apply should be the one put into practice. An operator in a moving automobile, observing crops and seeing that they are recorded on the crop meter, is not likely to make efficient use of a complicated definition.

Mathematical Appendix

The frequency distribution of fields in a county is assumed to be

$$df_A = \frac{a^{b+1}}{\Gamma(b+1)} e^{-aA} A^b dA$$

If the probability of encountering a field on a mile of route is proportional to the square root of its area, the frequency distribution of the fields on the routes is

$$df_{A_r} = \frac{a^{b+3/2}}{\Gamma(b+3/2)} e^{-aA_r} A_r^{b+1/2} dA_r$$

The constants a and b depend upon the average size of the fields in the county and the coefficient of variation. Applying the method of moments, the values of the constants are found to be

$$a = \frac{1}{\bar{A}v^2}, \qquad b = \frac{1}{v^2} - 1$$

in which v^2 is expressed as a decimal fraction.

The average area of the fields in the county is,

$$\bar{A} = \frac{a^{b+1}}{\Gamma(b+1)} \int_0^\infty e^{-aA} A^{b+1} dA = \frac{b+1}{a}$$

The average area of the fields on the routes is

$$\bar{A}_r = \frac{a^{b+3/2}}{\Gamma(b+3/2)} \int_0^{\infty} e^{-aA_r} A_r^{\,b+3/2}\, dA_r = \frac{b+3/2}{a}$$

$$\frac{2(A_r - A)}{\bar{A}} = \frac{2}{\bar{A}}\left[\frac{b+3/2}{a} - \frac{b+1}{a}\right] = \frac{1}{a\bar{A}} = v^2$$

If the probability of getting a field of area A in a mile of route is $c_2\sqrt{A}$ and the total number of fields per square mile of area is n, the expected number of fields per mile of route is

$$n_r = \frac{c_2 n a^{b+1}}{\Gamma(b+1)} \int_0^{\infty} e^{-aA} A^{b+1/2} dA = \frac{c_2 n \Gamma(b+3/2)}{a^{1/2}\Gamma(b+1)}$$

If v is taken as equal to 0.48048, this expression reduces to

$$n_r = 0.97163\, c_2\, n \sqrt{\bar{A}}\ .$$

Under the assumption of a homogeneous distribution of fields, the variance of a relative frontage based on k miles of route was shown to be equal to

$$\frac{1}{k}(c_1^2 + c_3)c_2\ S(n_i A_i^{3/2})$$

in which n_i is the number of fields

ot area A_i per square mile. If $S(n_i) = n$, one obtains,

$$S(n_i A_i^{3/2}) = \frac{n a^{b+1}}{\Gamma(b+1)} \int_0^{\infty} e^{-aA} A^{b+3/2} dA = \frac{n\Gamma(b+5/2)}{a^{3/2}\Gamma(b+1)}$$

Taking v as equal to 0.48048, this expression reduces to,

$$S(n_i A_i^{3/2}) = 1.0838\ n\ \bar{A}^{3/2}$$

and the variance of the relative frontage reduces to

$$1.0838(1/k)\,(c_1^2 + c_3)c_2\, n\,\bar{A}^{3/2}.$$